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LINEAR AMPLIFIER DESIGN AND LINEAR INTEGRATED CIRCUITS

WILLIAM JOSEPH HOBLER, JR.

[redacted]

LINEAR AMPLIFIER DESIGN AND LINEAR INTEGRATED CIRCUITS

by

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Lieutenant Commander, United States Navy
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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

Through the utilization of the functional matrix description of linear integrated circuits, a design method is evolved from which the terminating admittances may be specified. These terminating admittances are optimized with respect to power gain for a stable device and optimized with respect to power gain for a specified stability level. The total input and output conductances are obtained for use in frequency response design. Finally the procedure is extended to include the case in which a not optimum conductance is forced on the designer.

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CHAPTER I

INTRODUCTION

The design of low power level circuitry can no longer be regarded as an exercise in stable bias design, impedance matching and feedback design. As more and more devices appear on the market, the selection of a device which will be stable under the specified circuit operating conditions becomes increasingly important and difficult. The high active device densities which result from the industry move toward miniaturization constrain the circuit designer to those designs which are compatible with this packaging and the thermal variations inherent to these high densities. The circuit designer must then select the proper device, configure that device to meet the performance specifications, prove the stability of the circuit and allow sufficient engineering time to permit proper packaging and thermal design.

Recently Linear Integrated Circuits (LICs) have become economically competitive with other low power devices in the frequency range D-C through 130 MHz. The designer, therefore, must consider LICs if only for the convenience of 135 db gain in a flatpack or an instrument amplifier with 70 to 90 db common mode rejection. The designer of military equipment can look toward increased reliability, decreased maintenance and decreased logistic support requirements which result from use of LICs. The designer of consumer electronics may select LICs because of the lower initial circuit cost, easier automation of the manufacturing process and the decrease in assembly operations which all reduce the manufacturing cost.

Although the introduction of LICs has increased the selection of devices, LICs provide the designer with the following distinct advantages:

1. Reduced size and weight
2. Reduced power drain
3. Improved reliability
4. Lower cost
5. Improved maintainability
6. Improved stability

It is the intention of this paper to justify briefly the first five of these advantages with indications of future trends in an effort to illuminate a broad range of system or product improvements accruing from Linear Integrated Circuits. The sixth advantage, stability, is to be just one portion of a fairly vigorous investigation of a circuit design technique which is comprehensive enough to include stability of the design.

SYSTEM OR PRODUCT IMPROVEMENT BY USE OF LINEAR INTEGRATED CIRCUITS

1. Reduced Size and Weight. The presently available size and weight reduction are most conveniently displayed by comparison of these parameters in Table I.[1]

TABLE I

COMPARISON OF WEIGHTS AND SIZES OF CONVENTIONAL AND MICROELECTRONIC CIRCUITS

Construction	Miniature Conventional	TO-5 Can	Integrated Circuit		Thin Film 1/4 x 1/4 x 0.025 substrata
			1/4 x 1/4 Flatpack	Chip	
Weight in oz.	0.055	0.35	0.0055	0.000035	0.002
Volume in in.^{3}	0.1	0.04	0.008	0.000016	0.008

Note, although the basic chip size represents a reduction in volume of 10,000, the package controls the volume and degrades this advantage to a 10-20 to one reduction. Hallman predicts, "By 1975, we may expect the overall size of avionics packaging to be approximately 1/30 the size of the equivalent 1965 package. However, the 1980 package will occupy only 1/75 the volume of the equivalent 1965 functional package."[2] These size reductions presume no increase in functional capabilities. However, increases in capabilities are indicated by a predicted 25 per cent increase in active device population on board a U.S. Navy destroyer in the period 1965 to 1970. Application of integrated circuits will reduce size and weight by 17 per cent during the same period.[3] Thus, in both comparable systems and increased capability systems, the size and weight of the electronics package will decrease through integration.

2. Reduced Power Drain. Whitelock[4] contends that 50 per cent to five per cent of the presently required power will be required for comparable systems when integrated circuitry is used. These figures are supported by a reduction from 21 watts to 3.9 watts due to the use of LICs in a PCM Multicoder by Burns and Foulke.[5] Similarly, the power requirements for a conventional versus an integrated circuit FM Telemetry Encoder was reduced by half.[6]

3. Improved Reliability. Hallman states, "Even so, we may expect that our principal reliability problem in 1980 will be one of devising ways to measure and/or predict mean-time-between-failure rates in excess of 1000 hours."[7] This problem exists today in an Ultra-Reliable Transceiver under evaluation by the U. S. Navy. In this program two six channel UHF transceivers have been produced with the design goal of optimum performance for two years with no maintenance, with a confidence interval of 98 per cent.[8] This is a mean-time-between-failures of 17,520 hours. "To approach this degree of reliability, an all-solid-state transceiver was mandatory along with maximum utilization of microelectronic techniques and redundant and adaptive circuitry."[9]

4. Lower Cost. Perhaps the most telling argument to support the lower cost contention comes from the highly profit-motivated consumer electronics industry. H. H. Scott described its decision to use integrated circuitry to replace discrete components as based purely on economic considerations.[10] However, this lower cost is reflected

throughout the spectrum of system life. During initial construction, the integrated circuit can replace numbers of discrete components, thus reducing the production operations required, saving labor costs. The increased reliability increases the mean-time-between-repair causing a decrease in down time and repair cost. Additionally, trouble shooting circuitry may be included in some of the reduced volume where reduced down time and/or training or salary of repair personnel is a determining factor. In fact, it may become economical to discard a 135 db amplifier (costing $1.50) rather than troubleshoot it.

5. Improved Maintainability. "As the effective failure- or trouble-free life of subassemblies increases, the economic advantage of simply replacing defective subassemblies and discarding the faulty module becomes more attractive. Also, the skill levels required of maintenance personnel are reduced considerably."[11]

In summary, the benefits that are derived from microelectronic techniques - reliability, size, weight, maitainability, cost and power consumption - are real. "For all these reasons, circuit and system design must assume greater importance and must be applied with greater care. The time and money spent on optimizing circuit design will become a worthwhile investment... One result os such effort could be a more thorough and complete characterization of circuits than we have ever had in the past and in a systematic way."[12] Such complete characterization of circuits has been available to the circuit designer for some years. Systematic methods of applying these characterizations and describing circuit operation is also available. If current applications literature is to be used as an indicator, these techniques have not been broadly applied to Linear Integrated Circuits.

CHAPTER II

DEVICE AND CIRCUIT STABILITY

On of the most common and most exasperating difficulties in circuit design is the problem of oscillation. Oscillation is always caused by some form of feedback from the output to the input. Such feedback can be a result of the circuit interconnections either wired in or through stray capacitance and inductive coupling. Feedback may also occur through reverse transmission within the active device. Oscillation due to the reverse transmission will be referred to as device-caused oscillation.[13] The existence of internal feedback or reverse transmission is expressed by the fact that if the device is described by the "z", "y", "h", or "g" matrices that element having the subscript 12 is different from zero. Most active devices presently available have sufficient internal feedback to cause oscillation when properly terminated and operated at the circuits resonant frequency.[14] It can and will be shown that Linear Integrated Circuits possess sufficient internal feedback to support device-caused oscillation. Since the circuit may oscillate it is proper to begin an investigation of LIC application with a discussion of stability.

1. Characterization. Any discussion of active devices must begin with the characterization of the device. The characterization most suitable for the study of the performance of an LIC is a functional characterization from which the designer determines the potential of the circuit

including stability, sensitivity, gain, and optimum load and source impedances.[15] In characterization of LICs, it is not practical to separately characterize each component. Initially, it is not possible to reach every point in the circuit. If an equivalent circuit formulation is attempted, the stray capacitance and lead inductance form complex pi-networks around the active circuits which severely compound the characterization. Additionally, since a diffused resistor is, in fact, a distributed element, its equivalent circuit is a R-C transmission line and the equivalent circuit becomes untenable. To avoid the difficulties imposed by the above consideration, the functional two-port matrix characterization is used for LICs.[16]

The familiar two-port characterizations are the "h", "g", "z" and "y" parameters. These parameters implicitly but completely characterize the device. Any set of two-port parameters may be calculated from any other set so that the set of parameters actually used is determined by the designer. Since the device may be parameterized in any of four ways and an analysis must produce identical physical results for any set of parameters, analysis utilizing any parameter set is equally valid providing the chosen parameters are consistently applied. Therefore, an analysis using one set of parameters provides a general series of design relationships.[17] Here it is well to note that the following analysis will produce certain "invariant" properties. This use of the term invarient implies invariant in form under the transformations between the common two-port parameters.

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Although it is true that Mason's unilateral power gain U is invariant to the way in which the terminals are paired in a standard transistor,[18] since the substrate of an integrated circuit transistor is always connected to the common terminal, any change in the configuration changes the internal circuit. This alteration of the internal circuit changes the power gain.[19] Therefore, whenever the device under consideration is an LIC, the invariance deals with the use of consistent sets of parameters and not with the shifting of terminal pairs.

2. Definitions of Stability. Any useful active device may be categorized, with respect to stability, as inherently stable or potentially unstable. The device is considered inherently stable if no combination of passive terminations can cause oscillation. However, if the internal feedback within the active device is sufficient to support device-caused oscillation, the device is considered potentially unstable.[20] With reference to Figure 1, if there can be found some combination of y_s and y_L which will cause oscillations, the active device is potentially unstable.

$$y_s = g_s + j b_s$$
$$y_L = g_L + j b_L$$
$$y_{11} = g_{11} + j b_{11}$$
$$y_{22} = g_{22} + j b_{22}$$

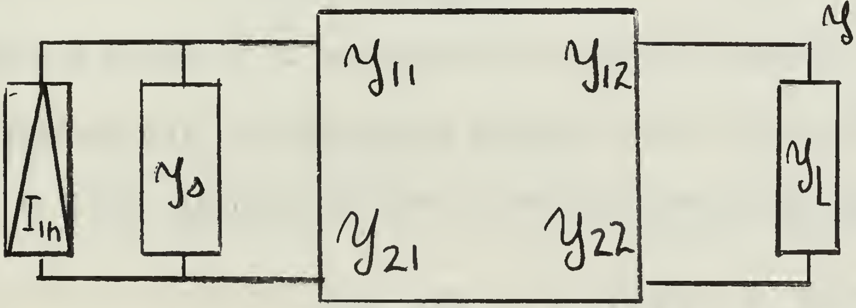

FIGURE 1
Terminated Two-port

Rollett introduced the invariant stability factors, k and K defined as

$$k = \frac{2g_{11}g_{22} - Re(y_{12}y_{21})}{|y_{12}y_{21}|} \qquad K = \frac{2G_{11}G_{22} - Re(y_{12}y_{21})}{|y_{12}y_{21}|} \tag{1}$$

$$\text{where } G_{11} = g_s + g_{11} \text{ and } G_{22} = g_L + g_{22} \tag{2}$$

A device is stable if $k \geq 1$ and $g_{11}g_{22} \geq 0$. Similarly, a circuit is stable if $K \geq 1$ and $G_{11}G_{22} \geq 0$.[21] Note that if no combination of y_s and y_L can cause K to become less than unity and $G_{11}G_{22} < 0$ the device is inherently stable. Bahrs casts these two expressions into equivalent forms given below:

$$g_{11}g_{22} > M/2(1 + \cos\theta) \tag{3}$$

$$\text{where } M = |y_{12}y_{21}|; \quad \theta = \arg(y_{12}y_{21}) \tag{4}$$

In this form the device is inherently stable if the above inequality is valid. If G_{11} and G_{22} are defined as above a potentially unstable device may be made inherently stable by neutralization or by loading the device sufficiently with large source and load conductances. The combination of these conductances and the active device is inherently stable if

$$G_{11}G_{22} > M/2(1 + \cos\theta) \tag{5}$$

where M and θ are as defined in equations (4).

Note that the parameters involved in measuring the stability are functions of frequency. Any particular device may be potentially unstable over any portion of its usable frequency range. Thus, the statement that an active device is potentially unstable is really incomplete unless the frequency range over which this instability exists is included in the statement.[22]

The expression $G_{11}G_{22}$ (M/2)(1 + Cos θ) suggests the use of a new stability factor for the terminated two-port.

$$K \triangleq \frac{2 G_{11} G_{22}}{M(1 + \cos\theta)} \tag{6}$$

where

$$\begin{aligned} M &\triangleq |y_{12} y_{21}| \\ \theta &\triangleq \arg(y_{12} y_{21}) \end{aligned} \tag{7}$$

The terminated two-port will be stable if $K \geq 1$. Similarly, an inherent stability factor may be defined by

$$K_i \triangleq \frac{2 g_{11} g_{22}}{M(1 + \cos\theta)} \tag{8}$$

where M and θ are defined by (7) above.

By this definition, if $K_1 \geq 1$ no combination of passive source and load terminations can support device-caused oscillations. Conversely, if K_i is less than unity, a suitable pair of passive terminations exists which can cause oscillation. Therefore, the device is inherently stable if $K_i \geq 1$ and potentially unstable if $K_i < 1$. If sufficient external conductance is added to the ports of a potentially unstable device, K the actual stability factor may be made greater than unity and the circuit stable.

The actual stability factor K is identical with the factor K of Stern and of Bahrs. It is not invariant under the transformations of the two-port parameters.[23]

CHAPTER III

AMPLIFIERS USING INHERENTLY STABLE DEVICES

For two-port networks which require power at the input port, as LICs do, it is more significant to measure performance in terms of the power in the circuit. The ratio of power delivered to the load to the power available from the source is defined as transducer gain, and given the symbol G_T.

$$G_T = \frac{\text{Power to the load}}{\text{Available source power}} = \frac{P_{out}}{P_{avail}} \tag{9}$$

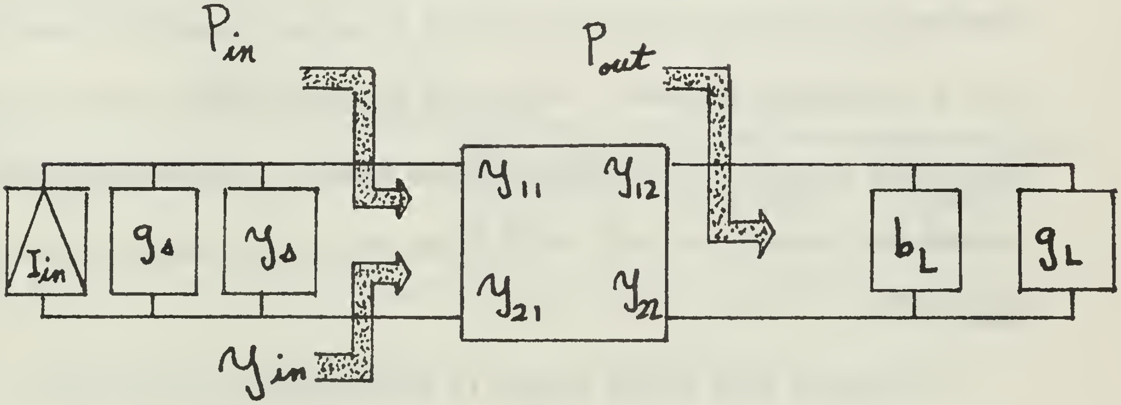

FIGURE 2
Power in a Terminated Two-Port

Unfortunately, transducer gain is a function of the source impedance, the device parameters, and the load impedance. It is, therefore, convenient to consider another quantity which provides an upper bound on transducer gain and is easier to calculate. Power gain is the ratio of the power supplied to load to the power into the two-port and is given the symbol G_p.[24]

$$G_p = \frac{\text{Power delivered to load}}{\text{Power into the twoport}} = \frac{P_{out}}{P_{in}} \tag{10}$$

The transducer gain may be expressed as

$$G_T = \frac{P_{out} \quad P_{in}}{P_{in} \quad P_{avail}} \tag{11}$$

For an inherently stable device, P_{in}/P_{avail} is always less than or equal to unity. The maximum value of unity can be achieved by conjugately matching y_s to y_{in}. At a given frequency or in a narrow band of frequencies, this can always be done by using a suitable transformer and a susceptive element. Thus, the transducer gain is the same as the power gain for an inherently stable device. The design problem reduces to finding the load termination which will maximize the power gain.[25]

The power gain of the device is independent of the source admittance and is expressible as[26]

$$G_p = \frac{|y_{21}|^2 \, Re(y_L)}{Re(y_{11} - y_{12} y_{21}/[y_{22} + y_L]))\,|y_{22} + y_L|^2} \tag{12}$$

After some manipulation equation (12) may be rewritten as

$$G_p = \left|\frac{y_{21}}{y_{12}}\right| \frac{M \; g_L}{g_{11}(G_{22}^2 + B_{22}^2) - M G_{22} \cos\theta - M B_{22} \sin\theta} \tag{13}$$

Now by defining a normalized transducer gain G_{TN} and a normalized power gain G_{PN} such that

$$G_T = \left|\frac{y_{21}}{y_{12}}\right| G_{TN} \quad \text{and} \quad G_p = \left|\frac{y_{21}}{y_{12}}\right| G_{PN} \tag{14}$$

so that

$$G_{PN} = \frac{G_P}{|y_{21}/y_{12}|} \quad \text{and} \quad G_{TN} = \frac{G_T}{|y_{21}/y_{12}|} \tag{15}$$

Since $|y_{21}/y_{12}|$ is a constant of the device, a maximum value of G_{PN} yields a maximum value of G_p or G_T. Now from equations (13) and (15) the normalized power gain is

$$G_{PN} = \frac{M(G_{22} - g_{22})}{\left\{g_{11}G_{22} - MG_{22}\operatorname{Cos}\theta - \dfrac{M^2 \operatorname{Sin}^2\theta}{4g_{11}}\right\} + g_{11}\left(B_{22} - \dfrac{M \operatorname{Sin}\theta}{2g_{11}}\right)^2} \tag{16}$$

Since $B_{22} = b_{22} + b_L$ occurs only in the second term of the denominator the value of B_{22} which maximizes G_{PN} is obviously

$$B_{22} - \frac{M \operatorname{Sin}\theta}{2g_{11}} = 0 \tag{17}$$

or the optimum value of B_{22}, $B_{22_{opt}}$

$$B_{22_{opt}} = \frac{M \operatorname{Sin}\theta}{2g_{11}} \tag{18}$$

Thus the optimum output susceptance is independent of the load conductance. Substitution of $B_{22_{op}}$ into equation (16) and rearranging yields

$$G_{PN} = \frac{2\left[\left\{\frac{2g_{11}G_{22}}{M} - \cos\theta\right\} - \left\{\frac{2g_{11}g_{22}}{M} - \cos\theta\right\}\right]}{\left\{\frac{2g_{11}G_{22}}{M} - \cos\theta\right\}^2 - 1} \tag{19}$$

Now if an inherent stability figure is defined as ϕ_i where

$$\phi_i \triangleq \frac{2g_{11}g_{22}}{M} - \cos\theta \tag{20}$$

and a partial stability figure is defined as ϕ_p where

$$\phi_p \triangleq \frac{2g_{11}G_{22}}{M} - \cos\theta \tag{21}$$

then equation (19) may be written as

$$G_{PN} = \frac{2(\phi_p - \phi_i)}{\phi_p^2 - 1} \tag{22}$$

The inherent stability figure was defined as a function of the device parameters whereas the partial stability figure is a function of the device parameters and the load conductance. Equation (22) represents the normalized power gain for any load conductance for the optimum load susceptance. The optimum load conductance may be calculated from that partial stability figure which maximizes G_{PN}. Differentiating equation (22) and equating the result to zero yields

$$\phi_p^2 - 2\phi_p \phi_i + 1 = 0 \tag{23}$$

From the definitions of the two stability figures, it is obvious that ϕ_p is always greater than ϕ_i, thus the solution for the optimum partial stability figure is $\phi_{p_{opt}}$, where

$$\phi_{p_{opt}} = \phi_i + \sqrt{\phi_i^2 - 1} \tag{24}$$

If this value of ϕ_p is substituted into the expression of G_{PN} the maximum value of G_{PN}, G_{PN_o}, results

$$G_{PN_o} = \frac{1}{\phi_i + \sqrt{\phi_i^2 - 1}} \tag{25}$$

The value of normalized power gain for various values of ϕ_i is plotted in Figure 3. Since for all values of ϕ_i less than unity the power is unbounded at $\phi_p = 1$, the device is potentially unstable. For ϕ_i greater than unity the gain is finite for all ϕ_p going through a maximum value at $\phi_{p_{opt}}$ at a value of G_{PN_o}.

By expression (3) Bahrs had defined the stability relation $g_{11}g_{22}$ $(M/2)(1 + \text{Cos}\theta)$ which may be written as[27]

$$\frac{2 g_{11} g_{22}}{M} - \text{Cos}\,\theta > 1 \tag{26}$$

then the condition for inherent stability may just as validly be written as $\phi_i > 1$, since $\phi_i \triangleq \frac{2g_{11}g_{22}}{M} - \cos\theta$. Similarly, the condition for potential instability may be written as $\phi_i < 1$. In an analogous manner a stability figure for the terminated two-port is defined as ϕ

$$\phi \triangleq \frac{2G_{11}G_{22}}{M} - \cos\theta \tag{27}$$

At this point the problem of amplifier design is completed since the load susceptance may be determined from equation (18) and the load conductance may be determined from $\phi_{p_{opt}}$ and equation (21). Then the input admittance need only be conjugately matched. The computation load may be reduced by recognizing that equations (20), (21) and (24) and the optimum value of conductance $g_{L_{opt}} = G_{22_{opt}} - g_{22}$ yields

$$\frac{2g_{11}g_{L_{opt}}}{M} = \sqrt{\phi_i^2 - 1} \tag{28}$$

Therefore, G_{PN_O} and equation (28) are functions of the inherent stability figure and may be plotted as functions of ϕ_i, as is done in figure 3.[28]

ILLUSTRATIVE EXAMPLE

To illustrate the design procedure using an inherently stable device, an RCA differential amplifier, CA 3028, was selected and an operating frequency of 10.7 mega Hz used as center frequency. For

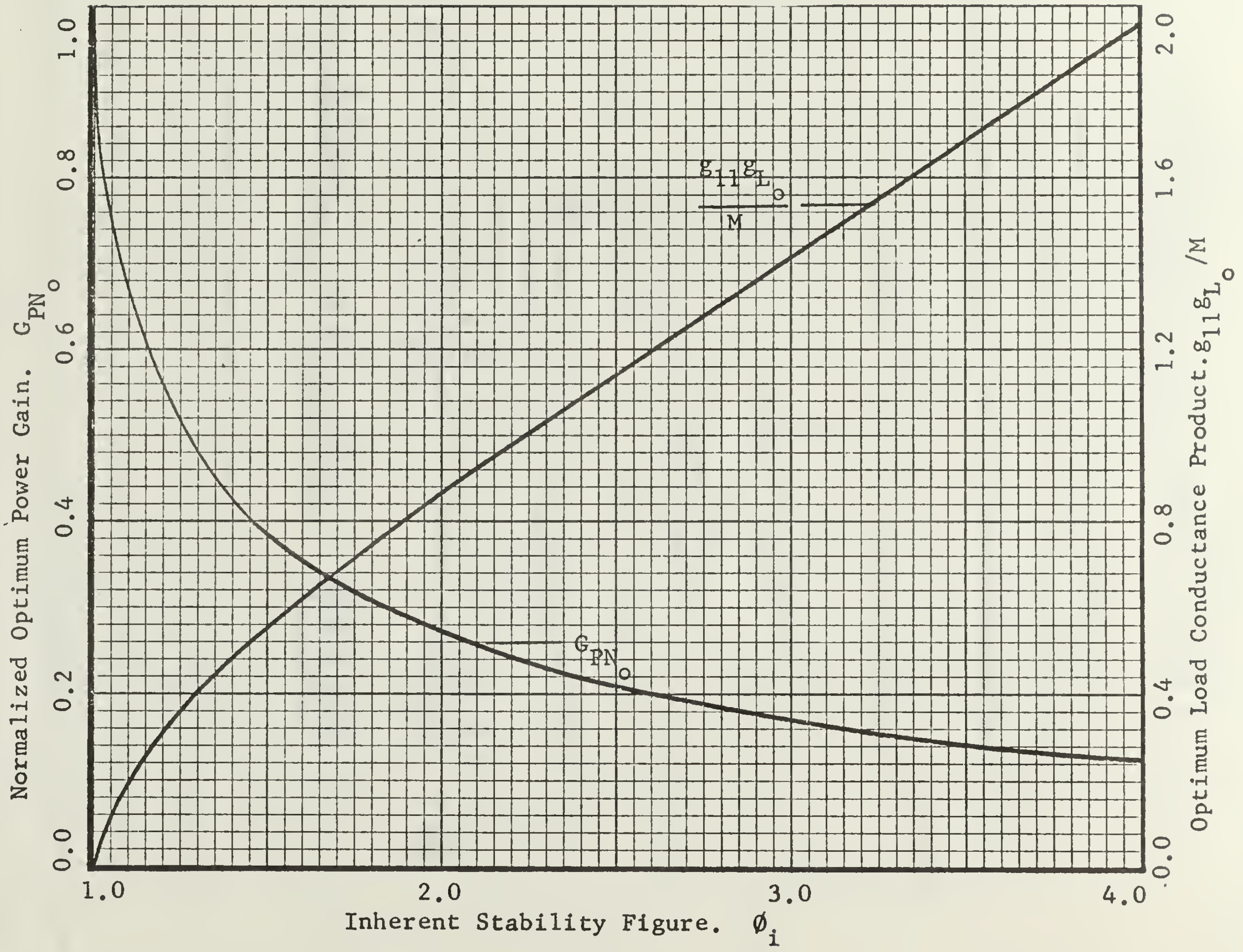

Figure 3. Normalized optimum power gain and optimum conductance product as functions of the inherent stability figure.

this condition the specification sheet lists the following admittance parameters.[29]

$y_{11} = .5 + j.5$ mmhos $\quad y_{22} = .04 + j.23$ mmhos

$y_{12} = .01 - j.0002$ mmhos $\quad y_{21} = -37 + j.5$ mmhos

1. Calculate intermediate but individually useful values.

   a. $M = y_{12}y_{21} = .37 \times 10^{-6}$ mhos^2

   b. $\theta = \arg(y_{12}y_{21}) = 178.0^{\circ}$

      (1) $\cos\theta = -.99942$

      (2) $\sin\theta = .03403$

2. Calculate the inherent stability figure ϕ_i

$$\phi_i = \frac{2g_{11}g_{22}}{M} - \cos\theta = \frac{2 \cdot .5 \cdot .04}{.37} + .99942 \qquad (20)$$

$$= \underline{1.0074} \quad \text{inherently stable}$$

3. From figure 3 enter with ϕ_i and read the following

   a. $G_{PN_o} = .935$

   b. $g_{11}g_{L_o'}/M = .03$

4. Calculate the optimum load terminations

   a. $g_{L_o} = (M/g_{11}).03 = 2.22 \times 10^{-3}$ mhos

b. Since $b_{L_o} = G_{22_{opt}} - b_{22}$ from equation (18)

$$b_{L_o} = \frac{M \sin\theta}{2g_{11}} - b_{22} \tag{29}$$

$$= \underline{-.23 \text{ mmhos}}$$

5. Calculate the optimum source terminations. Lathi, by a straight forward calculation shows that the conjugate match at the input terminals for a device terminated with the optimum admittance is as follows:[30]

$$\text{a.} \quad g_{s_o} = \left\{\frac{g_{11}}{g_{22}}\right\} g_{L_o} = \frac{.5}{.04} \cdot 2.22 \times 10^{-5} \tag{30}$$

$$= \underline{27.75 \times 10^{-5} \text{ mhos}}$$

$$\text{b.} \quad b_{s_o} = \frac{M \sin\theta}{2g_{11}} - b_{11} \tag{31}$$

$$= \underline{-.4874 \times 10^{-3} \text{ mhos}}$$

6. Calculate the stage power gain.

$$G = \left|\frac{y_{21}}{y_{12}}\right| G_{PN_o} = \frac{37}{.01} \; .935 = \underline{3459.5} \tag{14}$$

The design problem is thus complete for the stated conditions. A source addmittance of $y_s = .2775 - j.4874$ milli mhos and a load admittance $y_L = .0222 - j.23$ milli mhos should yield a maximum stage power gain of 3459.5 or 35.4 db. In this real world this design should have been stopped at the calculation of ϕ_i. Figure 3 amply illustrates the sensitivity of the stage gain to changes in ϕ_i, when ϕ_i is close to unity. It does not take too much experience in the electronics field to conceive of parameter drifts which would shift this device into the potentially unstable category. An immediate attempt to design a less sensitive and more stable circuit would be the addition of some conductance at the input and output ports producing larger g_{11} and g_{22}. While it is true that ϕ is increased, it is not true that the maximum power gain for a specific level of stability may be achieved by this arbitrary procedure. A more rigorous and satisfying procedure is obtained by treating this device as a potentially unstable device, selecting a circuit stability figure ϕ and design as in the following section.

CHAPTER IV

AMPLIFIERS USING POTENTIALLY UNSTABLE DEVICES

The transducer gain of a generalized terminated two port device may be expressed as G_T, where

$$G_T = \frac{4|y_{21}|^2 g_s g_L}{|(y_s + y_{11})(y_L + y_{22}) - y_{12} y_{21}|^2} \tag{32}$$

By defining

$$G_{11} \triangleq g_s + g_{11} \qquad B_{11} \triangleq b_s + b_{11}$$

$$G_{22} \triangleq g_L + g_{22} \qquad B_{22} = b_L + b_{22}$$

$$y_{12} y_{21} \triangleq a + jb$$

the transducer gain may be written as

$$G_T = \frac{4|y_{21}|^2 g_s g_L}{|(G_{11} + jB_{11})(G_{22} + jB_{22}) - (a + jb)|^2} \tag{33}$$

Optimization of the source and load susceptances involves finding those values of b_s and b_L which minimize the denominator of equation (33). As a matter of convenience this denominator is written as

$$D = |(G_{11} G_{22} - B_{11} B_{22} - a) + j(B_{11} G_{22} + B_{22} G_{11} - b)|^2 \tag{34}$$

$$= (G_{11} G_{22} - B_{11} B_{22} - a)^2 + (B_{11} G_{22} + B_{22} G_{11} - b)^2 \tag{35}$$

The partial derivatives of D with respect to B_{11} and B_{22} are as follows

$$\frac{\partial D}{\partial B_{11}} = -2(G_{11}G_{22} - B_{11}B_{22} - a)B_{22} + 2(G_{11}B_{22} + G_{22}B_{11} - b)G_{22} \tag{36}$$

$$\frac{\partial D}{\partial B_{22}} = -2(G_{11}G_{22} - B_{11}B_{22} - a)B_{11} + 2(B_{11}G_{22} + G_{11}B_{22} - b)G_{11} \tag{37}$$

and at the minimum D

$$\frac{\partial D}{\partial B_{11}} = \frac{\partial D}{\partial B_{22}} = 0 \tag{38}$$

This yields

$$(G_{11}G_{22} - B_{11}B_{22} - a)B_{22} = (G_{11}B_{22} + G_{11}B_{22} - b)G_{22} \tag{39}$$

$$(G_{11}G_{22} - B_{11}B_{22} - a)B_{11} = (G_{11}B_{22} + G_{22}B_{11} - b)G_{11} \tag{40}$$

Dividing equation (40) by equation (39) yields

$$\frac{B_{11}}{G_{11}} = \frac{B_{22}}{G_{22}} \tag{41}$$

Equation (41) states that to maximize the gain of the stage the ratio of susceptance to conductance at the input and output ports should be equal. This ratio is to be defined as γ, that is

$$\gamma \triangleq \frac{B_{11}}{G_{11}} = \frac{B_{22}}{G_{22}} \tag{42}$$

Now equation (35) may be written

$$D = \left(G_{11}G_{22}(1+\gamma^2) - a\right)^2 + \left(2\gamma G_{11}G_{22} - b\right)^2 \tag{43}$$

and its partial derivative with respect to γ is

$$\begin{aligned}\frac{\partial D}{\partial \gamma} = -4G_{11}G_{22}\{G_{11}G_{22}(1-\gamma^2) - a) \\ + 4G_{11}G_{22}(2G_{11}G_{22} - b)\end{aligned} \tag{44}$$

This partial derivative must in turn be equal to zero at the minimum of D, which leads to the relation

$$\gamma^3 + \left\{1 + \frac{a}{G_{11}G_{22}}\right\}\gamma - \frac{b}{G_{11}G_{22}} = 0 \tag{45}$$

However, the formulation of a and b are

$$a = M \operatorname{Cos} \theta \qquad b = M \operatorname{Sin} \theta \tag{46}$$

and from the definition of stability figure Ø, (equation (27)

$$G_{11}G_{22} = \frac{M}{2}\left(\phi + \operatorname{Cos} \theta\right) \tag{47}$$

equation number (45) may be expressed as

$$\gamma^3 + \left\{1 + \frac{2 \operatorname{Cos} \theta}{\phi + \operatorname{Cos} \theta}\right\}\gamma - \frac{2 \operatorname{Sin} \theta}{\phi + \operatorname{Cos} \theta} \tag{48}$$

This cubic equation (48) is in a "normal" form whose roots are discussed in a number of sources; see for example Burington.[31]

Using equations (46) and (47) equation number (43) may be rewritten as

$$D = \left\{\frac{M}{2}(\phi + \cos\theta)(1-\gamma^2) - M\cos\theta\right\}^2 + \left\{2\frac{M}{2}(\phi + \cos\theta)\gamma - M\sin\theta\right\}^2$$

which reduces to

$$D = M^2\left\{\left[\frac{1}{2}(\phi + \cos\theta)(1-\gamma^2) - \cos\theta\right]^2 + \left[\gamma(\phi + \cos\theta) - \sin\theta\right]^2\right\} \qquad (49)$$

Examination of equation (49) particularly the last squared term reveals that if θ is in the first two quadrants, D is minimized by some positive value of γ . Further, it is noted that the equation (49) is minimized under the condition that the sign of γ is identical with the sign of sinθ, that is, γ is positive for θ in the first two quadrants and γ is negative for θ in quadrants 3 and 4. Therefore, examination of both (48) and (49) leads to

$$\gamma(\phi, \theta) = -\gamma(\phi, \theta) \qquad (50)$$

$$D(\phi, \theta) = -D(\phi, \theta) \qquad (51)$$

Investigation of equation (48) results in the conclusion that for θ in the first quadrant the roots consist of one positive real root and two imaginary roots. Since only physically realizable solutions are to be considered, the positive root is the desired value of γ . For θ in the second quadrant, the roots of (48) consist of one positive root and two negative roots, of which the positive root will minimize D.

In summary then, for a given value of ϕ and for θ lying between 0 and 180 degrees, D minimum may be found by solving equation (48) for the one positive real root and substituting this root into equation (49). For other values of θ the relations (50) and (51) will yield the desired D.

With this minimum value of D the transducer gain may be written as

$$G_T = \frac{4|y_{21}|^2 g_s g_L}{D} \tag{52}$$

which is $|y_{21}/y_{12}|$ times the normalized transducer gain G_{TN} of equation (14).

$$G_{TN} = \frac{4 M g_s g_L}{D} \tag{53}$$

Since the minimum value of D has been found the remaining factor under control of the circuit designer is the product $g_s g_L$. This product must be maximized to give the maximum gain but this maximization is subject to the stability criteria.

$$\phi = \frac{2 G_{11} G_{22}}{M} - \cos\theta \tag{27}$$

that is, the product $g_s g_L$ is restricted to the condition

$$G_{11} G_{22} = \frac{M}{2}(\phi + \cos\theta) = (g_s + g_{11})(g_L + g_{22}) \tag{54}$$

However, by the definitions of equation (33), g_s and g_L may be written as

$$g_s = G_{11} - g_{11} \qquad g_L = G_{22} - g_{22}$$

and the product as

$$g_s g_L = (G_{11} - g_{11})(G_{22} - g_{22}) \tag{55}$$

Finally by defining a dimensionless ratio of the total terminal conductance to the device conductance G/g as r where

$$r_{11} \triangleq \frac{G_{11}}{g_{11}} \qquad r_{22} \triangleq \frac{G_{22}}{g_{22}} \tag{56}$$

the product may be written as

$$g_s g_L = g_{11} g_{22} (r_{11} - 1)(r_{22} - 1) \tag{57}$$

which is a maximum when the partial derivatives are equal and zero.

$$\frac{\partial (g_s g_L)}{\partial r_{11}} = \frac{\partial (g_s g_L)}{\partial r_{22}} = 0 \tag{58}$$

By the equation (57) $g_s g_L$ is seen to be symmetrical with respect to r_{11} and r_{22}, which, of course, means that the maximum will be reached when these two ratios are equal. Therefore, the subscript will be dropped from r and the product written as

$$g_s g_L = g_{11} g_{22} (r - 1)^2 \tag{59}$$

and

$$G_{11} G_{22} = r^2 g_{11} g_{22} = \frac{M}{2}(\phi + \cos\theta) \tag{60}$$

which results in the ratio r as a function of the device constants and the stability figure

$$r^2 = \frac{M(\theta + \cos\theta)}{2 g_{11} g_{22}} = \frac{\phi + \cos\theta}{\phi_i + \cos\theta} \tag{61}$$

where ϕ_i is the inherent stability figure defined by equation (20).

The normalized transducer gain may now be written in terms of equations (59) and (61).

$$G_{TN} = \frac{4 M g_{11} g_{22} \left\{ \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\}^2}{D}$$

$$= \frac{g_{11} g_{22} \cdot 2(\phi + \cos\theta) \left\{ 1 - \sqrt{\frac{\phi_i + \cos\theta}{\phi + \cos\theta}} \right\}^2}{M^2 \left[\left\{ \frac{1}{2}(\phi + \cos\theta)(1 - \gamma^2) - \cos\theta \right\}^2 + \left\{ \gamma(\phi + \cos\theta) - \sin\theta \right\}^2 \right]}$$

$$= \frac{\left\{ 1 - \sqrt{\frac{\phi_i + \cos\theta}{\phi + \cos\theta}} \right\}^2 \left\{ 2(\phi + \cos\theta) \right\}}{\left[\frac{1}{2}(\phi + \cos\theta)(1 - \gamma^2) - \cos\theta \right]^2 + \left[\gamma(\phi + \cos\theta) - \sin\theta \right]^2}$$

(62)

When written in this form G_{TN} is seen to consist of two groupings of factors.

1. $\left\{1-\sqrt{\dfrac{\phi_i+\cos\theta}{\phi+\cos\theta}}\right\}^2$ a function of g_{11}, g_{22}, g_s, g_L, $\arg(y_{12}, y_{21})$ and ϕ.

2. F, the remaining factors in equation (62) is defined below and is a function of the transmission admittances and the stability figure. (y_{12}, y_{21}, ϕ)

$$F \triangleq \frac{2(\phi+\cos\theta)}{\left[\frac{1}{2}(\phi+\cos\theta)(1+\gamma^2)-\cos\theta\right]^2+\left[\gamma(\phi+\cos\theta-\sin\theta\right]^2} \tag{63}$$

With this background the normalized transducer gain maximum may be expressed as

$$G_{TN_0} = F\left\{1-\sqrt{\frac{\phi_i+\cos\theta}{\phi+\cos\theta}}\right\}^2 \tag{64}$$

A set of optimum terminating conductances may now be obtained from equation (56).

$$\frac{G_{11}}{g_{11}} = r = \sqrt{\frac{\phi+\cos\theta}{\phi_i+\cos\theta}} = \frac{g_s+g_{11}}{g_{11}}$$

that is

$$g_s = g_{11}\left\{\sqrt{\frac{\phi + \text{Cos}\,\Theta}{\phi_i + \text{Cos}\,\Theta}} - 1\right\} \tag{65}$$

Similarly

$$g_L = g_{22}\left\{\sqrt{\frac{\phi + \text{Cos}\,\Theta}{\phi_i + \text{Cos}\,\Theta}} - 1\right\} \tag{66}$$

With these conductances, equation (42) yields the remaining susceptances.[32]

$$\frac{B_{11}}{G_{11}} = \frac{B_{22}}{G_{22}} = \frac{b_s + b_{11}}{g_s + g_{11}} = \frac{b_L + b_{22}}{g_L + g_{22}} \tag{42}$$

$$b_s = (g_s + g_{11})\gamma - b_{11} \qquad b_L = (g_L + g_{22})\gamma - b_{22} \tag{67}$$

With this determination of the terminating admittances and the gain of the amplifier the design is in essence complete. The effort required of the designer has thus far not been reduced because the evaluation of F, γ , the gain and conductances require solutions to transcendental equations. However, this approach has separated the arguments of the problem into grouping which can be conveniently computer programmed and the results tabulated. Initially γ was computed by Bahrs and used by Lathi. This paper has reformulated the work of these two into a consistent notation. Therefore, γ has been recomputed and tabled as a function of $\arg(y_{12}y_{21})$, and ϕ the stability figure. Similarly, F, the gain factor has been tabled as a function of these two variables. See Tables II and III. The other transcendental factor of interest

$$\left\{1 - \sqrt{\frac{\phi_i + \operatorname{Cos}\theta}{\phi + \operatorname{Cos}\theta}}\right\}^2 = \left\{1 - \frac{1}{r}\right\}^2$$

is plotted in Figure 4 as a function of the ratio r. Figure 5 plots γ for several values of ϕ to illustrate the smooth behavior of this function, although Table II steps ϕ through integral values and θ in five degree steps linear interpolation between any combination of values is a valid operation. Similarly, Figure 6 plots F for several values of ϕ to support the validity of linear interpolation between values of F.

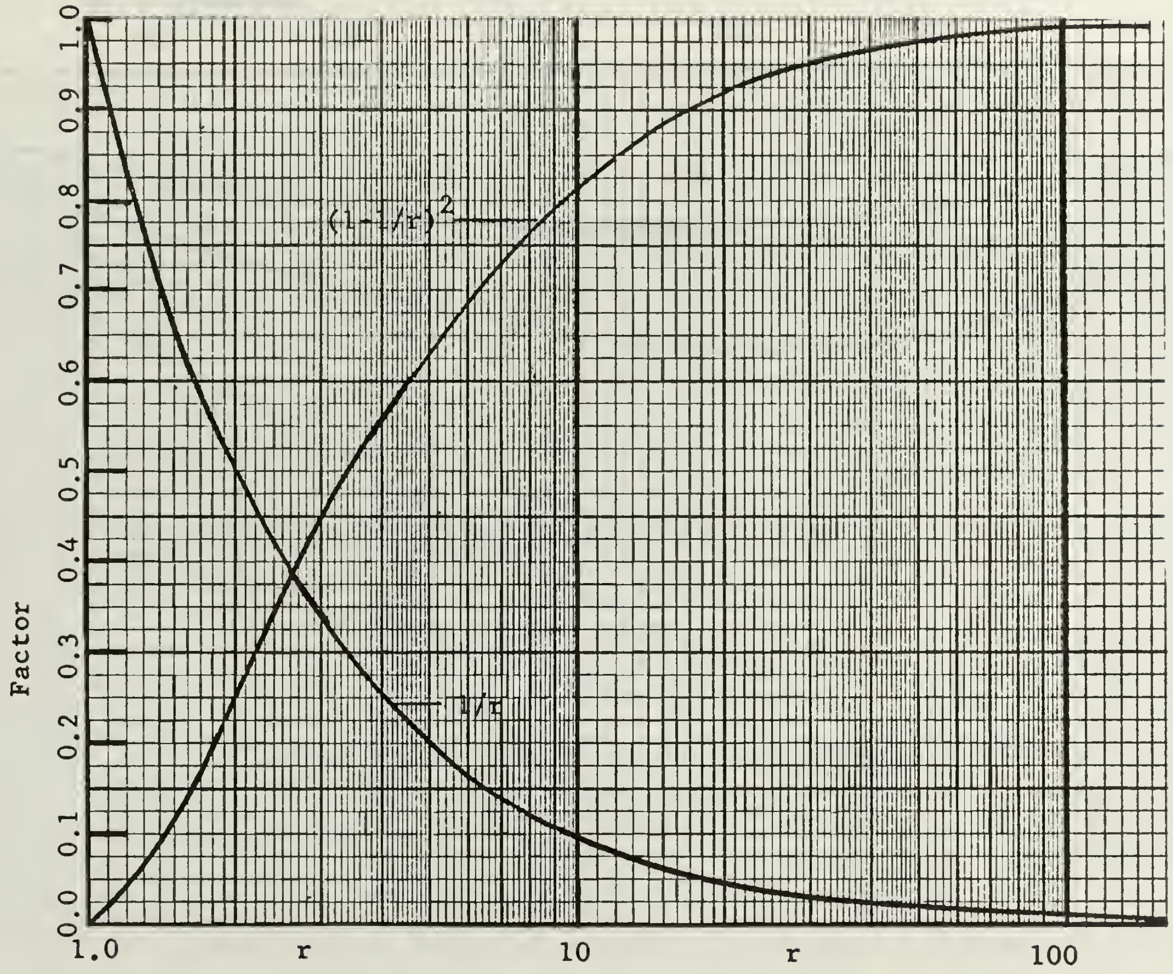

Figure 4. The factors 1/r and $(1-1/r)^2$ as functions of r.

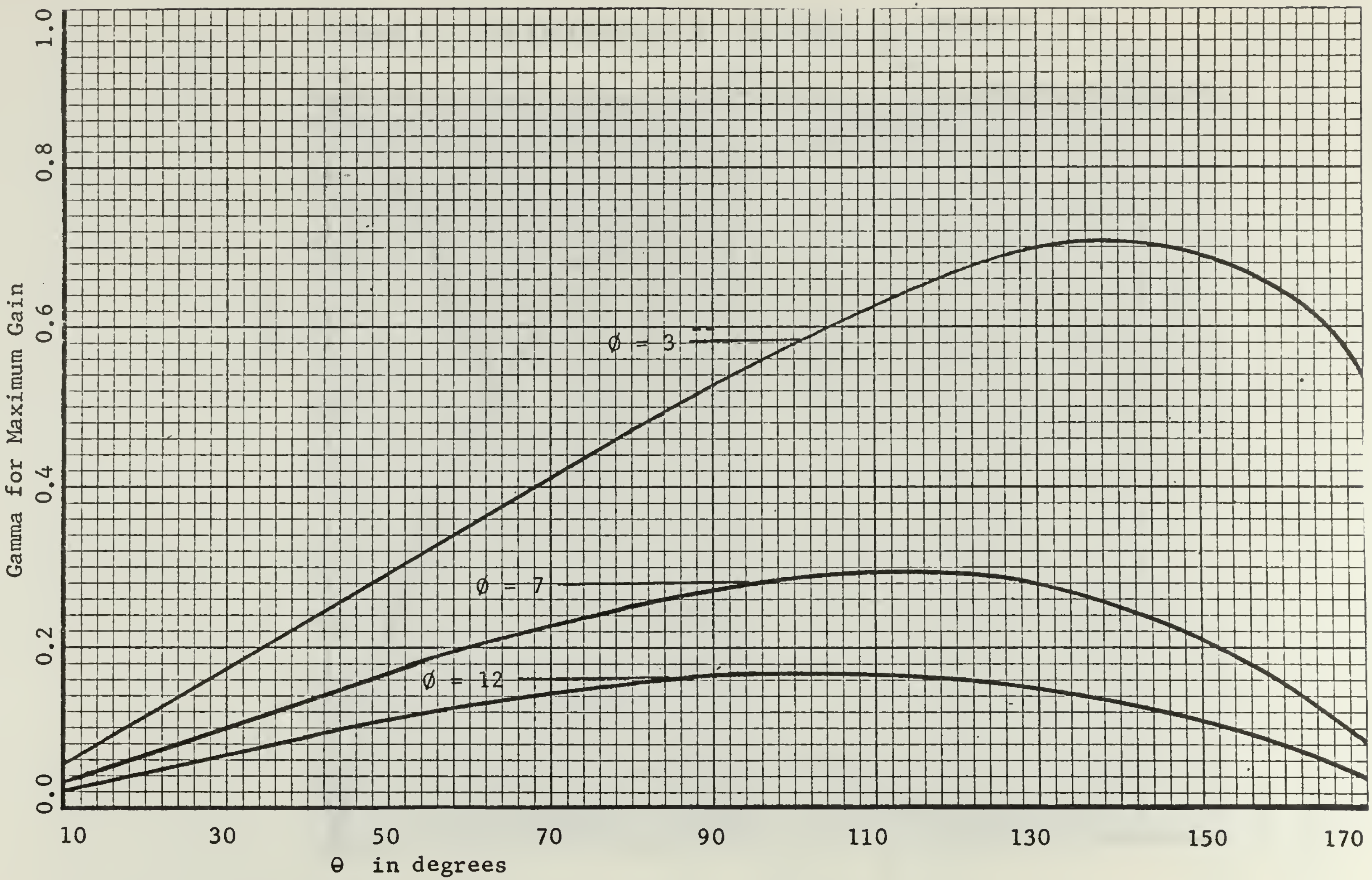

Figure 5. The value of Gamma required to maximize gain as a function of θ, for several values of the stability figure ∅.

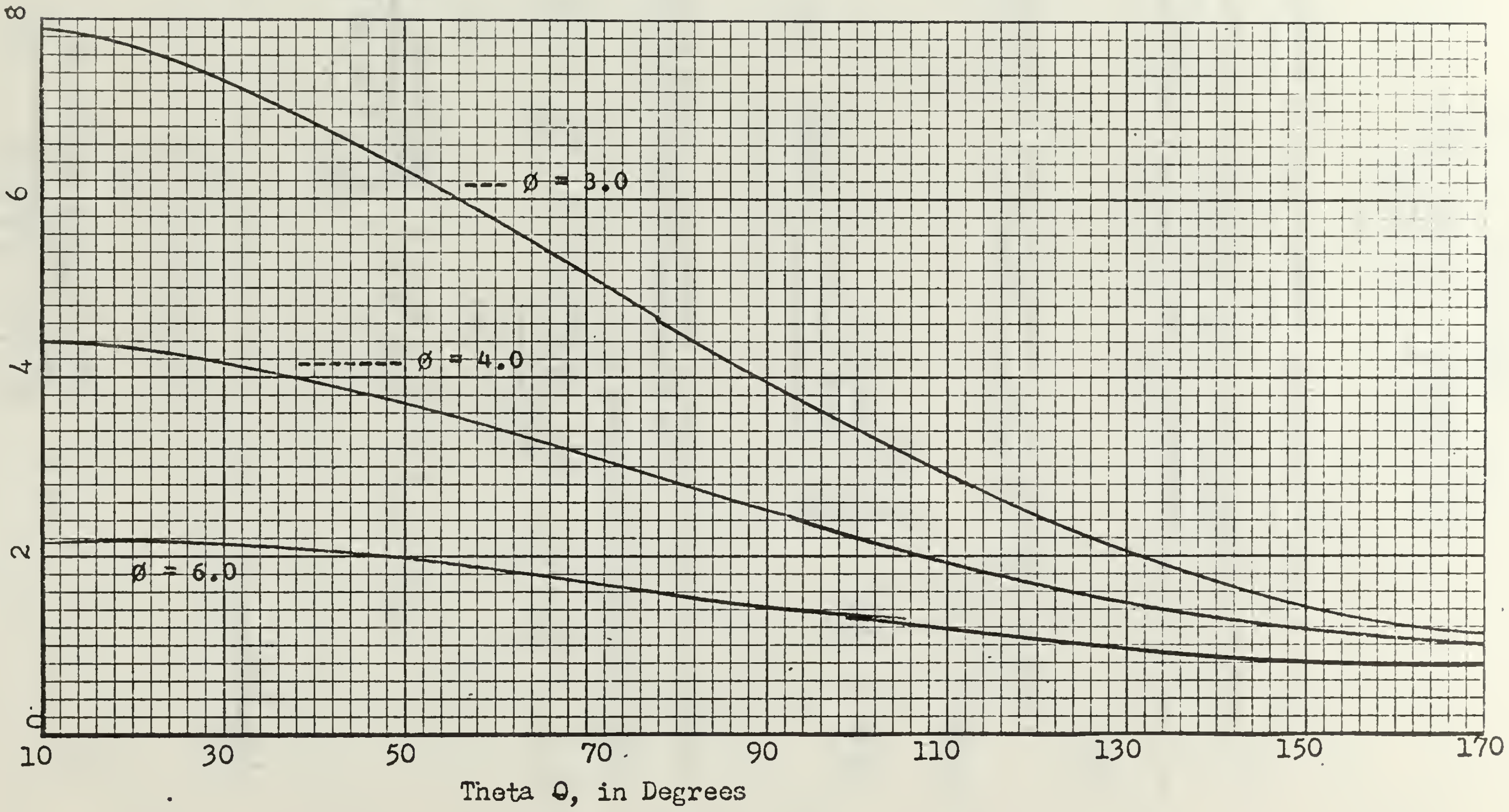

Figure 6. The gain Factor, F, as a function of Theta for Several Values of the Stability Figure Phi. Ø.

CHAPTER V

THE TERMINAL PROPERTIES OF THE AMPLIFIER STAGE

One of the more important considerations of amplifier design is that of bandpass characteristics. The bandpass is, of course, a function of the total terminal admittance at the input and output terminals of the stage.

The input admittance of a two port which is terminated at the output by y_L is

$$y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + y_L} \tag{68}$$

to which the input terminating admittance y_s must be added to give the total admittance at the input Y_{in}

$$Y_{in} = y_s + y_{11} - \frac{y_{12}y_{21}}{y_{22} + y_L}$$

$$= g_s + g_{11} + j(b_s + b_{11}) - \frac{M(\mathrm{Cos}\,\theta + j\,\mathrm{Sin}\,\theta}{g_{22} + g_L + j(b_{22} + b_L)}$$

yielding

$$Y_{in} = G_{11}(1 - j\gamma) - \frac{M(\mathrm{Cos}\,\theta + j\,\mathrm{Sin}\,\theta)}{G_{11}(1 + j\gamma)} \tag{69}$$

Separation of the real and imaginary parts of Y_{in} yields G_{in} and B_{in} as

$$G_{in} = G_{11} - \frac{M(\cos\theta + \gamma \sin\theta)}{G_{22}(1+\gamma^2)} \tag{70}$$

and

$$B_{in} = G_{11}\gamma - \frac{M(\sin\theta - \gamma\cos\theta)}{G_{22}(1+\gamma^2)} \tag{71}$$

Considering first G_{in}. By defining a function of γ, and θ, call it $R(\gamma, \theta)$ as

$$R(\gamma,\theta) \triangleq \frac{\cos\theta + \gamma\sin\theta}{(1+\gamma^2)} \tag{72}$$

the input conductances may be expressed as

$$g_{in} = g_{11} - \frac{M}{G_{22}} R(\gamma,\theta) \tag{73}$$

and

$$G_{in} = G_{11} - \frac{M}{G_{22}} R(\gamma,\theta) \tag{74}$$

Similar arguments allow the output conductances to be expressed as

$$g_{out} = g_{22} - \frac{M}{G_{11}} R(\gamma,\theta) \tag{75}$$

$$G_{out} = G_{22} - \frac{M}{G_{11}} R(\gamma,\theta) \tag{76}$$

The total stage terminal susceptances should be zero at the point of maximum gain. If B_{in} is to be zero, then equation (71) is zero

$$G_{11}\gamma - \frac{M(\sin\theta - \gamma\cos\theta)}{G_{22}(1+\gamma^2)} = 0$$

or

$$G_{11}G_{22}(\gamma - \gamma^3) = M(\sin\theta - \gamma\cos\theta)$$

yielding

$$G_{11}G_{22}(\gamma - \gamma^3) = a - \gamma b \tag{77}$$

or $$\gamma^3 + \left(1 + \frac{a}{G_{11}G_{22}}\right)\gamma - \frac{b}{G_{11}G_{22}} = 0 \tag{45}$$

The susceptance condition which yields zero total terminal susceptance is, therefore, identical with the condition of maximum transducer gain. Under these conditions the terminal susceptance is known to be zero. However, under non-optimum conditions, the susceptance can most conveniently be calculated by defining another function of γ and θ, call it I(γ , θ), where

$$I(\gamma,\theta) \triangleq \frac{\sin\theta - \gamma\cos\theta}{1+\gamma^2} \tag{78}$$

Allowing the input and output susceptances are expressible as

$$b_{in} = b_{11} - \frac{M}{G_{22}} I(\gamma,\theta) \tag{79}$$

$$B_{in} = B_{11} - \frac{M}{G_{22}} I(\gamma,\theta) = G_{11}\gamma - \frac{M}{G_{22}} I \tag{80}$$

$$b_{out} = b_{22} - \frac{M}{G_{11}} I(\gamma, \theta) \tag{81}$$

$$B_{out} = G_{22}\gamma - \frac{M}{G_{11}} I(\gamma, \theta) \qquad 82)$$

These formulations of susceptance and conductance are convenient by virtue of the computer generated tables of $R(\gamma, \theta)$ and $I(\gamma, \theta)$. See Tables IV and V. As with γ and F, R and $I(\gamma, \theta)$ are well behaved functions so that linear interpolation between tabulated values is a valid operation.

ILLUSTRATIVE EXAMPLE

The RCA integrated circuit CA 3028 may be used in a cascode configuration. The specification (op. cit., p. 8) lists the cascode y parameters at 10.7 MHz as

$y_{11} = (0.6 + j1.6)$m mhos $= 1.66 \angle 75.1^\circ$ mmhos

$y_{12} = (0.003 - j0.0)$ mmhos $= .0003 \angle 0.0^\circ$ mmhos

$y_{21} = (99 - j\,18)$ mmhos $= 100.8 \angle -10.3^\circ$ mmhos

$y_{22} = (0.0006 + j0.08)$ mmhos $= .08 \angle 90^\circ$ mmhos

1. Calculate the intermediate values

$M = \left| y_{12} y_{21} \right| = 0.0302 \times 10^{-6}$

$\arg(y_{12}, y_{21}) = \theta = -10.3^\circ$

$\cos\theta = 0.984 \qquad \sin\theta = -0.1788$

2. The inherent stability figure $\emptyset_i$

$$\emptyset_i = \frac{2g_{11}g_{22}}{M} - \cos\theta$$

$$= \frac{2(0.6 \times 10^{-3})(0.6 \times 10^{-6})}{0.0302 \times 10^{-6}} - 0.984$$

$= -0.960$ A POTENTIALLY UNSTABLE DEVICE

3. Choosing $\emptyset = 3.0$

   a. From Table II $\gamma_{\max} = 0.05819$

   b. From Table III $F = 7.92925$

   c. From Table IV $R(\gamma, \theta) = 0.99101$

4. Calculate the gain of the unit.

a. $$r = \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} = \sqrt{\frac{3 + 0.984}{-0.960 + 0.984}}$$

$$= 12.88$$

From Figure 4 $(1 - 1/r)^2 = 0.851$

b. The normalized stage gain from equation (64)

$$G_{TN_o} = F\left(1 - \frac{1}{r}\right)^2 = 7.92925(0.851)$$

$$= 6.74$$

c. Equation (14) yields gain

$$G_{TN} = \left|\frac{y_{21}}{y_{12}}\right| G_{TN_o} = \frac{100.6}{0.0003}(6.74)$$

$$= \underline{2.358 \times 10^6}$$

$$= \underline{63.72 \text{ db}}$$

5. The terminating admittances are calculated

a. Source conductance, equation (65)

$$g_s = g_{11}\left\{\sqrt{\frac{\phi + \cos\theta}{\phi_r + \cos\theta}} - 1\right\} = g_{11}(r - 1)$$

$$= 0.6 \times 10^{-3}(12.88 - 1) = \underline{7.16 \times 10^{-3} \text{ mhos}}$$

b. Source susceptance, equation (67)

$$b_s = (g_s + g_{11})\gamma - b_{11}$$

$$= \{(7.12 + 1.6)(0.0582) - 1.6\} \times 10^{-3}$$

$$= \underline{-1.15 \times 10^{-3} \text{ mhos}}$$

c. Load conductance, equation (66)

$$g_L = g_{22}(\Lambda - 1) = 0.006 \times 10^{-3}(11.88)$$

$$= \underline{7.12 \times 10^{-6} \text{ mhos}}$$

d. Load susceptance, equation (67)

$$b_L = (g_L + g_{22})\gamma - b_{22}$$

$$= \{(0.00712 + 0.0006)(0.0582) - 0.08\} \times 10^{-3}$$

$$= \underline{-0.08 \times 10^{-3}} \text{ mhos}$$

6. The terminal conductances

a. Source conductance, equation (74)

$$G_{in} = G_{11} - \frac{M}{G_{22}} R(\gamma, \theta)$$

$$= 7.72 \times 10^{-3} - \frac{0.0302}{7.72}(0.99101)$$

$$= \underline{3.95 \times 10^{-3} \text{ mhos}}$$

b. Load conductance, equation (76)

$$G_{out} = G_{22} - \frac{M}{G_{11}} R(\gamma, \theta)$$

$$= 7.72\times10^{-6} - \frac{0.0302\times10^{-6}}{7.72\times10^{-3}}(0.99101)$$

$$= \underline{3.95\times10^{-6}\,\text{mhos}}$$

This circuit was mechanized with some interesting results. Initially some difficulty is encountered in synthesizing the admittance

$y_L = 7.12\text{x}10^{-6} - j80\text{x}10^{-6}$.

The parallel inductance required is

$L_p = \dfrac{1}{80 \times 10^6 \times 6.28 \times 10.7 \times 10^6} = 185\mu$H while the parallel resistance is $r_p = \dfrac{1}{7.12 \times 10^{-6}} = 141 \times 10^3$ohms.

The Q of the inductance required for a parallel circuit is rather high. A similar but not as severe problem exists in the input circuitry. The problem is circumvented at both parts of using equivalent an series circuits for a load and a parallel series circuit in the input to match a 600 ohm signal generator.

The test rig was wired and the source and load impedances measured. Several adjustments were required to obtain the required paramters. The final circuit used is shown in Figure 7. All capacitance units are microfarads, both inductances measured at 10.7 MHz, and all resistor values are listed as their nominal values. With the device

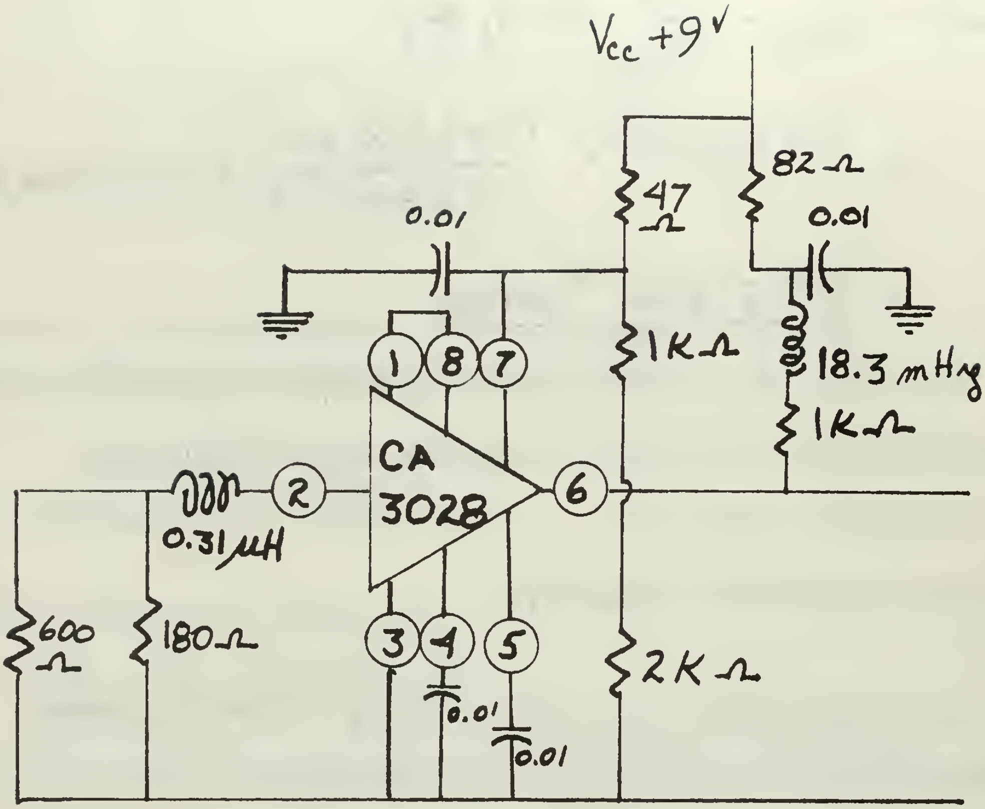

Figure 7
Cascode Amplifier with Optimum Terminations

removed, the r resistance from terminal 2 and terminal 6 to AC ground were 139 and 1020 ohms respectively. The calculated optimum values are 137 and 1105 ohms.

The voltage applied was 10mv peak. A clipped output caused a decrease of the input to 500 microvolts peak that is 353 x 10^{-6} volts rms. For a matched impedance, this voltage level results in an input power of 0.18 x 10^{-8} watts. Two integrated circuits were tested under these identical conditions. The output voltages observed were 1.41

and 1.38 volts rms which yields a power of 3.97×10^{-3} watts and 3.7×10^{-3} watts. The transducer gain P_{in}/P_{out} is then 2.2×10^{6} and 2.11×10^{6}. These values compare very favorably with the calculated 2.36×10^{6}.

Granted that the circuit configuration is impractical in form, however, the results and verification of this design approach justify the use of the configuration.

CHAPTER VI

NON-OPTIMUM LOAD AND SOURCE CONDUCTANCES

The conductances determined by equations (65) and (66) may be either not obtainable or not desirable by virtue of interstage network considerations. The effect of the departure of g_s and g_L from the optimum values is analyzed as follows:

The optimum value of G_{11} is

$$G_{11_{OPT}} = g_{11}\sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} \qquad (56)$$

and

$$G_{22_{opT}} = g_{22}\sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} \qquad (56)$$

However, let the actual value of G_{11}

$$G_{11} \triangleq \frac{1}{p} G_{11_{opt}} \qquad (83)$$

where p is a measure of the departure of G_{11} from the optimum value $G_{11_{opt}}$. For a given value of ϕ, the product $G_{11}G_{22}$ must be constant, that is

$$G_{11} G_{22} = G_{11_{opt}} G_{22_{opt}} \qquad (84)$$

Therefore

$$G_{22} = p\,G_{22opt}$$

These relations result in the values of g_s and g_L as

$$g_s = \left\{ \frac{1}{p} \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\} g_{11} \tag{86}$$

$$g_L = \left\{ p \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\} g_{22} \tag{87}$$

But from equation (62)

$$G_{TN} = \frac{2F}{M(\phi + \cos\theta)}\, g_s\, g_L \tag{62}$$

$$= \frac{2F}{M(\phi + \cos\theta)} \left\{ \frac{1}{p} \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\} \left\{ p \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\} g_{11}\, g_{22} \tag{88}$$

However, G_{TN_0} is

$$G_{TN_0} = \frac{2F}{M(\phi + \cos\theta)} \left\{ \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\}^2 g_{11}\, g_{22} \tag{89}$$

A measure of gain degradation may be defined as H

$$H \triangleq \frac{G_{TN}}{G_{TN_0}}$$

$$= \frac{\left\{ p \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\} \left\{ \frac{1}{p} \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\}}{\left\{ \sqrt{\frac{\phi + \cos\theta}{\phi_i + \cos\theta}} - 1 \right\}^2} \tag{90}$$

$$H = \frac{\left\{ p - \gamma\sqrt{\frac{\phi_i + \cos\theta}{\phi + \cos\theta}} \right\} \left\{ \frac{1}{p} - \gamma\sqrt{\frac{\phi_i + \cos\theta}{\phi + \cos\theta}} \right\}}{\left\{ 1 - \gamma\sqrt{\frac{\phi_i + \cos\theta}{\phi + \cos\theta}} \right\}^2} \tag{91}$$

Equation (91) is symmetrical with respect to p or $\frac{1}{p}$, that is, $H(\frac{1}{p}) = H(p)$ which shows that H is identical if $G_{22} = pG_{22_{opt}}$ or $G_{22} = \frac{1}{p} G_{22_{opt}}$.

The normalized transducer gain may now be written as

$$G_{TN} = F H \left\{ 1 - \gamma\sqrt{\frac{\phi_i + \cos\theta}{\phi + \cos\theta}} \right\} \tag{92}$$

Table VI tabulates H as a function of $\frac{\phi_i + \cos\theta}{\phi + \cos\theta}$ and p.

It can be seen that when this ratio is small the loss in transducer gain is small. For p = 5 and $\frac{\phi_i + \cos\theta}{\phi + \cos\theta} = .0025$, the loss in gain is only 18 per cent. A great deal of flexibility in design is possible for small values of $\frac{\phi_i + \cos\theta}{\phi + \cos\theta}$.

CHAPTER VII

CONCLUSION

This thesis was motivated by the introduction of linear integrated circuits into the devices available to the circuit design engineer. As soon as an application procedure was defined, the thesis dropped all contact with any specific active device. In fact, the original study of this subject referred to vacuum tubes and transistors as active devices. Linear integrated circuits, because of their complex structure, force the circuit designer away from any particular device-oriented model to the functional model of two port parameters. The analysis of circuit performance using these parameters is of great value for several reasons.

a. A two port parameter analysis is independent of the type of active device used.

b. Since all device response is compared within the same analytical framework, the design engineer more rapidly assimilates the response differences of each individual device.

c. The use of only one design method facilitates application of optimization techniques across the broad range of electronic devices.

d. The use of two port parameters and this design technique permit transforming from any given sen of parameters to any more convenient parameter set.

Objectively one must note a lack of consistent schemes of specifying devices used by the device manufacturers. Within the same industrial organization, technical specification vary in scheme and completeness. The vacuum tubes may use the transconductance and input/output capacitance; transistors, the h parameters; integrated circuits may utilize curves of the y parameters, curves of differential gain, etc., or incomplete g parameters. This proliferation of specifications forces either a transformation of parameters or, for a more thorough design, measurement of parameters.

The impact of linear integrated circuits into the electronics discipline is, I am sure, beyond our furthest imagining. The reduction of physical size resulting from LIC application makes feasible packaging of more functions per unit volume. The increase in available gain for a given cost and volume encourages sacrifice of a greater percentage of gain toward improvement of linearity or stability or, for that matter, any engineering trade off required by a particular application. As the precision of these devices is increased, their application to small, extremely fast analog computers of great accuracy is sure to emerge as a real time problem solver of high speed technological which are presently beyond the capability of our fastest digital computers and the algorithms used therein.

In the research required by this thesis, the lack of application of optimum design techniques to solid state circuit design problems is more and more impressive. The mean-time-between-failures of active devices is now measurable in terms of decades. The design of equipment utilizing these low failure rate components must optimize performance over component response change with lifetime. The statistical nature of these response changes forces the circuit designer to consider statistical circuit analysis, that is, to optimize design. Similarly, as modern society becomes more and more dependent on reliable accurate operation of a variety of electronic devices, it becomes more and more important that the performance of this equipment be optimized. The electronic circuit and electronic equipment designer, therefore, must learn and utilize the optimum design techniques of the control systems engineer.

THETA	PHI = 1.5	PHI = 2.0	PHI = 2.5	PHI = 3.0	PHI = 3.5	PHI = 4.0
10.0	0.07770	0.06989	0.06351	0.05819	0.05370	0.04985
15.0	0.11681	0.10499	0.09534	0.08732	0.08054	0.07474
20.0	0.15622	0.14026	0.12727	0.11648	0.10738	0.09960
25.0	0.19605	0.17577	0.15932	0.14568	0.13420	0.12439
30.0	0.23639	0.21158	0.19151	0.17494	0.16100	0.14911
35.0	0.27737	0.24774	0.22389	0.20424	0.18776	0.17373
40.0	0.31909	0.28431	0.25645	0.23359	0.21446	0.19823
45.0	0.36169	0.32135	0.28923	0.26298	0.24109	0.22255
50.0	0.40528	0.35890	0.32223	0.29239	0.26761	0.24668
55.0	0.45000	0.39703	0.35545	0.32181	0.29398	0.27055
60.0	0.49600	0.43577	0.38890	0.35121	0.32016	0.29410
65.0	0.54341	0.47515	0.42255	0.38053	0.34607	0.31728
70.0	0.59240	0.51522	0.45638	0.40971	0.37166	0.33998
75.0	0.64311	0.55598	0.49034	0.43869	0.39682	0.36212
80.0	0.69572	0.59743	0.52437	0.46738	0.42145	0.38357
85.0	0.75038	0.63957	0.55838	0.49564	0.44541	0.40420
90.0	0.80726	0.68233	0.59226	0.52333	0.46856	0.42385
95.0	0.86650	0.72563	0.62583	0.55029	0.49070	0.44234
100.0	0.92824	0.76935	0.65893	0.57629	0.51162	0.45946
105.0	0.99256	0.81331	0.69129	0.60108	0.53107	0.47495
110.0	1.05950	0.85725	0.72262	0.62436	0.54875	0.48854
115.0	1.12902	0.90082	0.75255	0.64576	0.56431	0.49988
120.0	1.20094	0.94360	0.78063	0.66486	0.57735	0.50862
125.0	0.0	0.98501	0.80634	0.68117	0.58741	0.51429
130.0	0.0	1.02433	0.82902	0.69410	0.59394	0.51640
135.0	0.0	0.0	0.84795	0.70297	0.59628	0.51431
140.0	0.0	0.0	0.86224	0.70697	0.59368	0.50730
145.0	0.0	0.0	0.87085	0.70513	0.58515	0.49443
150.0	0.0	0.0	0.0	0.69622	0.56946	0.47449
155.0	0.0	0.0	0.0	0.67867	0.54490	0.44581
160.0	0.0	0.0	0.0	0.65023	0.50893	0.40592
165.0	0.0	0.0	0.0	0.60729	0.45726	0.35080
170.0	0.0	0.0	0.0	0.54295	0.38120	0.27330

TABLE II. THE VALUES OF GAMMA WHICH MAXIMIZE GAIN AS A FUNCTION OF THE STABILITY FIGURE AND TRANSMISSION ANGLE.

THETA	PHI = 5	PHI = 6	PHI = 7	PHI = 8	PHI = 9	PHI = 10
10.0	0.04360	0.03874	0.03485	0.03168	0.02903	0.02679
15.0	0.06533	0.05802	0.05218	0.04741	0.04344	0.04008
20.0	0.08698	0.07720	0.06939	0.06302	0.05771	0.05323
25.0	0.10852	0.09624	0.08644	0.07845	0.07181	0.06621
30.0	0.12992	0.11509	0.10328	0.09367	0.08569	0.07895
35.0	0.15113	0.13371	0.11987	0.10862	0.09928	0.09142
40.0	0.17212	0.15206	0.13615	0.12324	0.11255	0.10356
45.0	0.19284	0.17007	0.15207	0.13749	0.12544	0.11532
50.0	0.21324	0.18770	0.16757	0.15130	0.13788	0.12663
55.0	0.23325	0.20487	0.18258	0.16460	0.14981	0.13744
60.0	0.25280	0.22152	0.19702	0.17733	0.16117	0.14767
65.0	0.27182	0.23755	0.21082	0.18941	0.17187	0.15727
70.0	0.29021	0.25289	0.22389	0.20074	0.18185	0.16615
75.0	0.30787	0.26742	0.23613	0.21125	0.19100	0.17423
80.0	0.32468	0.28103	0.24743	0.22082	0.19925	0.18143
85.0	0.34050	0.29360	0.25768	0.22936	0.20649	0.18767
90.0	0.35519	0.30497	0.26674	0.23673	0.21261	0.19283
95.0	0.36856	0.31499	0.27446	0.24282	0.21751	0.19683
100.0	0.38042	0.32348	0.28070	0.24749	0.22105	0.19955
105.0	0.39054	0.33025	0.28527	0.25058	0.22311	0.20089
110.0	0.39866	0.33506	0.28798	0.25193	0.22356	0.20072
115.0	0.40450	0.33767	0.28863	0.25137	0.22225	0.19894
120.0	0.40773	0.33779	0.28698	0.24872	0.21903	0.19543
125.0	0.40795	0.33512	0.28280	0.24377	0.21376	0.19006
130.0	0.40474	0.32930	0.27579	0.23634	0.20628	0.18274
135.0	0.39756	0.31993	0.26569	0.22621	0.19645	0.17335
140.0	0.38582	0.30655	0.25216	0.21318	0.18415	0.16184
145.0	0.36874	0.28865	0.23490	0.19706	0.16927	0.14813
150.0	0.34541	0.26566	0.21360	0.17771	0.15176	0.13225
155.0	0.31464	0.23696	0.18798	0.15504	0.13163	0.11423
160.0	0.27495	0.20194	0.15790	0.12910	0.10898	0.09420
165.0	0.22452	0.16019	0.12342	0.10007	0.08405	0.07240
170.0	0.16151	0.11174	0.08494	0.06839	0.05720	0.04914

TABLE II. THE VALUES OF GAMMA WHICH MAXIMIZE GAIN AS A FUNCTION OF THE STABILITY FIGURE AND TRANSMISSION ANGLE.

THETA	PHI = 1.5	PHI = 2.0	PHI = 2.5	PHI = 3.0	PHI = 3.5	PHI = 4.0
10.0	78.97695	23.73335	12.32197	7.92925	5.71371	4.41170
15.0	77.71456	23.40397	12.17056	7.84173	5.65643	4.37116
20.0	75.97746	22.95009	11.96170	7.72090	5.57728	4.31512
25.0	73.79500	22.37877	11.69842	7.56840	5.47730	4.24427
30.0	71.20369	21.69879	11.38447	7.38630	5.35777	4.15948
35.0	68.24628	20.92047	11.02431	7.17702	5.22020	4.06179
40.0	64.97078	20.05540	10.62295	6.94331	5.06631	3.95236
45.0	61.42929	19.11622	10.18583	6.68816	4.89798	3.83246
50.0	57.67677	18.11632	9.71879	6.41477	4.71722	3.70348
55.0	53.76979	17.06955	9.22784	6.12648	4.52613	3.56685
60.0	49.76524	15.98991	8.71912	5.82670	4.32686	3.42405
65.0	45.71907	14.89126	8.19875	5.51883	4.12157	3.27658
70.0	41.68509	13.78708	7.67269	5.20621	3.91241	3.12591
75.0	37.71384	12.69014	7.14667	4.89210	3.70146	2.97350
80.0	33.85163	11.61234	6.62608	4.57955	3.49069	2.82073
85.0	30.13968	10.56444	6.11585	4.27142	3.28197	2.66892
90.0	26.61342	9.55596	5.62043	3.97031	3.07702	2.51930
95.0	23.30206	8.59499	5.14369	3.67852	2.87738	2.37298
100.0	20.22819	7.68815	4.68893	3.39805	2.68443	2.23097
105.0	17.40778	6.84054	4.25879	3.13059	2.49933	2.09414
110.0	14.85018	6.05576	3.85531	2.87749	2.32306	1.96324
115.0	12.55846	5.33593	3.47992	2.63976	2.15642	1.83891
120.0	10.52985	4.68177	3.13346	2.41815	2.00000	1.72164
125.0	0.0	4.09275	2.81621	2.21306	1.85424	1.61184
130.0	0.0	3.56723	2.52798	2.02466	1.71938	1.50979
135.0	0.0	0.0	2.26817	1.85289	1.59557	1.41569
140.0	0.0	0.0	2.03580	1.69748	1.48281	1.32969
145.0	0.0	0.0	1.82964	1.55803	1.38104	1.25186
150.0	0.0	0.0	0.0	1.43404	1.29016	1.18229
155.0	0.0	0.0	0.0	1.32495	1.21005	1.12107
160.0	0.0	0.0	0.0	1.23026	1.14068	1.06842
165.0	0.0	0.0	0.0	1.14961	1.08224	1.02472
170.0	0.0	0.0	0.0	1.08297	1.03535	0.99079

TABLE III. THE GAIN FACTOR F AS A
FUNCTION OF THE STABILITY FIGURE AND TRANSMISSION ANGLE.

THETA	PHI = 5	PHI = 6	PHI = 7	PHI = 8	PHI = 9	PHI = 10
10.0	2.98104	2.22759	1.76901	1.46285	1.24494	1.08238
15.0	2.95754	2.21221	1.75813	1.45475	1.23866	1.07737
20.0	2.92503	2.19091	1.74307	1.44352	1.22996	1.07043
25.0	2.88388	2.16393	1.72398	1.42928	1.21892	1.06161
30.0	2.83456	2.13156	1.70105	1.41216	1.20565	1.05101
35.0	2.77763	2.09415	1.67453	1.39235	1.19027	1.03872
40.0	2.71373	2.05208	1.64467	1.37003	1.17294	1.02487
45.0	2.64355	2.00581	1.61179	1.34542	1.15381	1.00957
50.0	2.56785	1.95580	1.57619	1.31876	1.13307	0.99296
55.0	2.48741	1.90255	1.53823	1.29028	1.11090	0.97520
60.0	2.40306	1.84657	1.49826	1.26026	1.08750	0.95643
65.0	2.31562	1.78839	1.45664	1.22896	1.06307	0.93683
70.0	2.22593	1.72855	1.41374	1.19665	1.03783	0.91656
75.0	2.13481	1.66757	1.36994	1.16360	1.01198	0.89577
80.0	2.04305	1.60597	1.32559	1.13008	0.98574	0.87465
85.0	1.95141	1.54425	1.28105	1.09637	0.95930	0.85336
90.0	1.86061	1.48289	1.23666	1.06271	0.93288	0.83205
95.0	1.77133	1.42233	1.19275	1.02935	0.90666	0.81088
100.0	1.68418	1.36300	1.14962	0.99654	0.88083	0.79002
105.0	1.59971	1.30529	1.10757	0.96450	0.85558	0.76960
110.0	1.51841	1.24955	1.06687	0.93342	0.83107	0.74976
115.0	1.44073	1.19610	1.02775	0.90353	0.80746	0.73064
120.0	1.36703	1.14523	0.99045	0.87498	0.78491	0.71236
125.0	1.29763	1.09719	0.95517	0.84796	0.76355	0.69505
130.0	1.23281	1.05222	0.92212	0.82263	0.74351	0.67880
135.0	1.17279	1.01052	0.89145	0.79913	0.72492	0.66373
140.0	1.11778	0.97229	0.86335	0.77760	0.70790	0.64994
145.0	1.06796	0.93772	0.83797	0.75818	0.69256	0.63751
150.0	1.02354	0.90700	0.81548	0.74100	0.67900	0.62653
155.0	0.98474	0.88032	0.79602	0.72618	0.66732	0.61709
160.0	0.95183	0.85792	0.77978	0.71384	0.65762	0.60925
165.0	0.92522	0.84005	0.76691	0.70409	0.64997	0.60308
170.0	0.90542	0.82699	0.75757	0.69705	0.64445	0.59863

TABLE III. THE GAIN FACTOR F AS A FUNCTION OF THE STABILITY FIGURE AND TRANSMISSION ANGLE.

	GAMMA					
THETA	0.05	0.10	0.15	0.20	0.25	0.30
10.0	0.99101	0.99225	0.98861	0.98032	0.96774	0.95129
15.0	0.97643	0.98199	0.98264	0.97855	0.97001	0.95741
20.0	0.95441	0.96425	0.96919	0.96932	0.96489	0.95624
25.0	0.92513	0.93918	0.94836	0.95272	0.95244	0.94779
30.0	0.88880	0.90696	0.92032	0.92887	0.93273	0.93213
35.0	0.84572	0.86783	0.88527	0.89795	0.90593	0.90938
40.0	0.79619	0.82210	0.84349	0.86020	0.87223	0.87971
45.0	0.74061	0.77012	0.79528	0.81589	0.83189	0.84334
50.0	0.67939	0.71227	0.74102	0.76538	0.78522	0.80055
55.0	0.61300	0.64900	0.68113	0.70905	0.73258	0.75167
60.0	0.54195	0.58080	0.61604	0.64731	0.67436	0.69707
65.0	0.46677	0.50817	0.54628	0.58066	0.61101	0.63717
70.0	0.38804	0.43167	0.47235	0.50958	0.54301	0.57241
75.0	0.30635	0.35189	0.39483	0.43462	0.47087	0.50330
80.0	0.22233	0.26944	0.31430	0.35636	0.39516	0.43036
85.0	0.13663	0.18493	0.23138	0.27538	0.31643	0.35414
90.0	0.04988	0.09901	0.14670	0.19231	0.23530	0.27523
95.0	-0.03725	0.01234	0.06091	0.10777	0.15237	0.19423
100.0	-0.12409	-0.07442	-0.02535	0.02242	0.06829	0.11174
105.0	-0.20999	-0.16062	-0.11142	-0.06311	-0.01631	0.02841
110.0	-0.29430	-0.24559	-0.19664	-0.14815	-0.10079	-0.05515
115.0	-0.37636	-0.32870	-0.28036	-0.23207	-0.18451	-0.13828
120.0	-0.45556	-0.40930	-0.36195	-0.31422	-0.26681	-0.22036
125.0	-0.53129	-0.48679	-0.44078	-0.39398	-0.34709	-0.30076
130.0	-0.60298	-0.56058	-0.51626	-0.47075	-0.42473	-0.37887
135.0	-0.67007	-0.63009	-0.58781	-0.54393	-0.49913	-0.45410
140.0	-0.73207	-0.69482	-0.65489	-0.61297	-0.56974	-0.52588
145.0	-0.78850	-0.75425	-0.71698	-0.67734	-0.63601	-0.59365
150.0	-0.83893	-0.80794	-0.77362	-0.73656	-0.69743	-0.65690
155.0	-0.88297	-0.85549	-0.82437	-0.79018	-0.75355	-0.71516
160.0	-0.92029	-0.89652	-0.86884	-0.83778	-0.80394	-0.76797
165.0	-0.95061	-0.93074	-0.90670	-0.87900	-0.84821	-0.81493
170.0	-0.97369	-0.95786	-0.93766	-0.91354	-0.88602	-0.85570

TABLE IV. R OF GAMMA AND THETA.

	GAMMA					
THETA	0.40	0.45	0.50	0.55	0.60	0.65
10.0	0.90885	0.88395	0.85731	0.82942	0.80073	0.77165
15.0	0.92194	0.90012	0.87627	0.85088	0.82442	0.79730
20.0	0.92802	0.90944	0.88856	0.86588	0.84184	0.81688
25.0	0.92703	0.91184	0.89409	0.87428	0.85285	0.83024
30.0	0.91899	0.90730	0.89282	0.87603	0.85737	0.83728
35.0	0.90395	0.89585	0.88475	0.87111	0.85537	0.83794
40.0	0.88203	0.87759	0.86995	0.85956	0.84685	0.83224
45.0	0.85341	0.85264	0.84853	0.84147	0.83189	0.82019
50.0	0.81828	0.82121	0.82065	0.81698	0.81060	0.80191
55.0	0.77693	0.78353	0.78652	0.78627	0.78314	0.77752
60.0	0.72967	0.73989	0.74641	0.74957	0.74972	0.74722
65.0	0.67685	0.69061	0.70062	0.70717	0.71059	0.71123
70.0	0.61888	0.63608	0.64949	0.65939	0.66606	0.66982
75.0	0.55620	0.57670	0.59343	0.60659	0.61645	0.62332
80.0	0.48929	0.51294	0.53284	0.54917	0.56216	0.57207
85.0	0.41865	0.44528	0.46820	0.48757	0.50358	0.51647
90.0	0.34483	0.37422	0.40000	0.42227	0.44118	0.45694
95.0	0.26838	0.30032	0.32876	0.35375	0.37541	0.39394
100.0	0.18989	0.22413	0.25501	0.28253	0.30679	0.32793
105.0	0.10996	0.14624	0.17932	0.20917	0.23584	0.25943
110.0	0.02919	0.06723	0.10226	0.13421	0.16309	0.18895
115.0	-0.05180	-0.01229	0.02443	0.05824	0.08910	0.11704
120.0	-0.13240	-0.09171	-0.05359	-0.01818	0.01443	0.04423
125.0	-0.21199	-0.17044	-0.13120	-0.09446	-0.06035	-0.02891
130.0	-0.28997	-0.24787	-0.20781	-0.17003	-0.13467	-0.10183
135.0	-0.36574	-0.32341	-0.28284	-0.24430	-0.20797	-0.17398
140.0	-0.43873	-0.39650	-0.35572	-0.31670	-0.27968	-0.24480
145.0	-0.50838	-0.46656	-0.42589	-0.38670	-0.34927	-0.31376
150.0	-0.57416	-0.53307	-0.49282	-0.45376	-0.41619	-0.38033
155.0	-0.63557	-0.59553	-0.55600	-0.51736	-0.47995	-0.44401
160.0	-0.69214	-0.65346	-0.61494	-0.57703	-0.54006	-0.50431
165.0	-0.74344	-0.70641	-0.66921	-0.63230	-0.59605	-0.56077
170.0	-0.78909	-0.75398	-0.71839	-0.68276	-0.64751	-0.61296

TABLE IV. R OF GAMMA AND THETA.

	GAMMA					
THETA	0.75	0.80	0.85	0.90	0.95	1.00
10.0	0.71363	0.68520	0.65742	0.63044	0.60435	0.57923
15.0	0.74243	0.71523	0.68849	0.66236	0.63695	0.61237
20.0	0.76557	0.73982	0.71432	0.68923	0.66471	0.64086
25.0	0.78289	0.75878	0.73471	0.71086	0.68741	0.66446
30.0	0.79426	0.77197	0.74951	0.72709	0.70488	0.68301
35.0	0.79957	0.77928	0.75860	0.73777	0.71698	0.69636
40.0	0.79881	0.78066	0.76192	0.74285	0.72362	0.70442
45.0	0.79196	0.77609	0.75945	0.74227	0.72476	0.70711
50.0	0.77909	0.76562	0.75119	0.73604	0.72038	0.70442
55.0	0.76028	0.74933	0.73722	0.72421	0.71052	0.69636
60.0	0.73569	0.72733	0.71763	0.70686	0.69526	0.68301
65.0	0.70550	0.69980	0.69259	0.68414	0.67470	0.66446
70.0	0.66995	0.66694	0.66227	0.65621	0.64900	0.64086
75.0	0.62929	0.62900	0.62691	0.62329	0.61837	0.61237
80.0	0.58384	0.58628	0.58678	0.58562	0.58303	0.57923
85.0	0.53395	0.53909	0.54219	0.54350	0.54325	0.54168
90.0	0.48000	0.48781	0.49347	0.49724	0.49934	0.50000
95.0	0.42240	0.43281	0.44099	0.44719	0.45163	0.45452
100.0	0.36158	0.37451	0.38516	0.39375	0.40049	0.40558
105.0	0.29800	0.31337	0.32640	0.33730	0.34629	0.35356
110.0	0.23216	0.24984	0.26515	0.27829	0.28946	0.29884
115.0	0.16455	0.18441	0.20188	0.21716	0.23042	0.24185
120.0	0.09570	0.11758	0.13708	0.15438	0.16963	0.18302
125.0	0.02611	0.04985	0.07124	0.09042	0.10755	0.12279
130.0	-0.04368	-0.01826	0.00485	0.02578	0.04466	0.06163
135.0	-0.11314	-0.08623	-0.06158	-0.03906	-0.01858	0.00000
140.0	-0.18173	-0.15354	-0.12753	-0.10361	-0.08168	-0.06163
145.0	-0.24894	-0.21969	-0.19252	-0.16736	-0.14415	-0.12279
150.0	-0.31425	-0.28416	-0.25604	-0.22985	-0.20553	-0.18301
155.0	-0.37718	-0.34647	-0.31761	-0.29058	-0.26534	-0.24184
160.0	-0.43723	-0.40614	-0.37676	-0.34910	-0.32314	-0.29883
165.0	-0.49396	-0.46272	-0.43305	-0.40496	-0.37847	-0.35355
170.0	-0.54692	-0.51578	-0.48604	-0.45775	-0.43093	-0.40558

TABLE IV. R OF GAMMA AND THETA.

	GAMMA					
THETA	1.10	1.15	1.20	1.25	1.30	1.35
10.0	0.53205	0.51001	0.48901	0.46902	0.45002	0.43197
15.0	0.56590	0.54406	0.52316	0.50320	0.48416	0.46602
20.0	0.59544	0.57396	0.55333	0.53355	0.51462	0.49652
25.0	0.62045	0.59949	0.57928	0.55984	0.54116	0.52324
30.0	0.64074	0.62046	0.60083	0.58187	0.56358	0.54598
35.0	0.65615	0.63671	0.61781	0.59946	0.58171	0.56457
40.0	0.66657	0.64812	0.63008	0.61250	0.59542	0.57885
45.0	0.67191	0.65459	0.63756	0.62088	0.60459	0.58874
50.0	0.67214	0.65608	0.64018	0.62453	0.60916	0.59414
55.0	0.66726	0.65257	0.63794	0.62342	0.60910	0.59502
60.0	0.65730	0.64410	0.63083	0.61757	0.60440	0.59137
65.0	0.64233	0.63073	0.61893	0.60703	0.59510	0.58322
70.0	0.62248	0.61256	0.60232	0.59186	0.58127	0.57063
75.0	0.59789	0.58972	0.58112	0.57219	0.56302	0.55370
80.0	0.56875	0.56240	0.55550	0.54816	0.54048	0.53256
85.0	0.53528	0.53080	0.52565	0.51996	0.51383	0.50736
90.0	0.49774	0.49516	0.49180	0.48781	0.48327	0.47830
95.0	0.45641	0.45575	0.45421	0.45194	0.44903	0.44560
100.0	0.41160	0.41287	0.41317	0.41263	0.41138	0.40951
105.0	0.36367	0.36685	0.36897	0.37018	0.37059	0.37031
110.0	0.31296	0.31803	0.32197	0.32492	0.32698	0.32828
115.0	0.25987	0.26680	0.27252	0.27718	0.28089	0.28376
120.0	0.20481	0.21353	0.22100	0.22733	0.23265	0.23707
125.0	0.14819	0.15864	0.16779	0.17575	0.18265	0.18859
130.0	0.09044	0.10255	0.11331	0.12284	0.13125	0.13866
135.0	0.03200	0.04567	0.05796	0.06899	0.07886	0.08768
140.0	-0.02668	-0.01155	0.00217	0.01461	0.02587	0.03604
145.0	-0.08516	-0.06869	-0.05363	-0.03987	-0.02732	-0.01588
150.0	-0.14300	-0.12531	-0.10903	-0.09406	-0.08031	-0.06768
155.0	-0.19974	-0.18097	-0.16359	-0.14752	-0.13268	-0.11896
160.0	-0.25496	-0.23525	-0.21691	-0.19987	-0.18404	-0.16934
165.0	-0.30824	-0.28774	-0.26858	-0.25069	-0.23400	-0.21843
170.0	-0.35918	-0.33805	-0.31821	-0.29961	-0.28218	-0.26586

TABLE IV. R OF GAMMA AND THETA.

	GAMMA					
THETA	1.45	1.50	1.55	1.60	1.65	1.70
10.0	0.39858	0.38317	0.36854	0.35468	0.34153	0.32905
15.0	0.43230	0.41667	0.40179	0.38765	0.37421	0.36142
20.0	0.46273	0.44699	0.43199	0.41768	0.40404	0.39104
25.0	0.48964	0.47392	0.45889	0.44452	0.43080	0.41768
30.0	0.51282	0.49724	0.48230	0.46799	0.45427	0.44114
35.0	0.53210	0.51678	0.50204	0.48789	0.47429	0.46124
40.0	0.54733	0.53238	0.51796	0.50408	0.49071	0.47784
45.0	0.55839	0.54393	0.52994	0.51643	0.50338	0.49080
50.0	0.56521	0.55134	0.53789	0.52485	0.51223	0.50002
55.0	0.56772	0.55456	0.54174	0.52928	0.51718	0.50544
60.0	0.56591	0.55355	0.54147	0.52968	0.51819	0.50701
65.0	0.55980	0.54833	0.53708	0.52604	0.51525	0.50472
70.0	0.54942	0.53894	0.52860	0.51841	0.50840	0.49859
75.0	0.53486	0.52545	0.51609	0.50683	0.49768	0.48866
80.0	0.51624	0.50796	0.49966	0.49139	0.48317	0.47502
85.0	0.49368	0.48660	0.47943	0.47221	0.46498	0.45776
90.0	0.46737	0.46154	0.45555	0.44944	0.44325	0.43702
95.0	0.43750	0.43297	0.42820	0.42325	0.41815	0.41295
100.0	0.40430	0.40110	0.39759	0.39383	0.38987	0.38574
105.0	0.36802	0.36618	0.36396	0.36142	0.35862	0.35559
110.0	0.32894	0.32847	0.32756	0.32626	0.32464	0.32274
115.0	0.28736	0.28826	0.28866	0.28862	0.28819	0.28743
120.0	0.24359	0.24586	0.24757	0.24878	0.24955	0.24994
125.0	0.19797	0.20159	0.20459	0.20704	0.20901	0.21054
130.0	0.15084	0.15578	0.16005	0.16373	0.16687	0.16954
135.0	0.10256	0.10879	0.11430	0.11918	0.12347	0.12724
140.0	0.05351	0.06097	0.06768	0.07371	0.07913	0.08398
145.0	0.00404	0.01268	0.02054	0.02769	0.03418	0.04008
150.0	-0.04545	-0.03570	-0.02675	-0.01855	-0.01102	-0.00412
155.0	-0.09460	-0.08381	-0.07384	-0.06464	-0.05614	-0.04829
160.0	-0.14303	-0.13128	-0.12037	-0.11024	-0.10084	-0.09210
165.0	-0.19037	-0.17775	-0.16598	-0.15500	-0.14476	-0.13520
170.0	-0.23627	-0.22287	-0.21033	-0.19859	-0.18759	-0.17728

TABLE IV. R OF GAMMA AND THETA.

	GAMMA					
THETA	1.80	1.85	1.90	1.95	2.00	2.05
10.0	0.30599	0.29532	0.28520	0.27557	0.26642	0.25772
15.0	0.33769	0.32668	0.31620	0.30622	0.29672	0.28765
20.0	0.36683	0.35556	0.34480	0.33454	0.32475	0.31540
25.0	0.39317	0.38172	0.37078	0.36032	0.35031	0.34074
30.0	0.41652	0.40498	0.39393	0.38335	0.37321	0.36349
35.0	0.43670	0.42516	0.41409	0.40346	0.39326	0.38347
40.0	0.45356	0.44211	0.43110	0.42051	0.41033	0.40053
45.0	0.46696	0.45568	0.44482	0.43435	0.42427	0.41455
50.0	0.47681	0.46580	0.45516	0.44489	0.43498	0.42541
55.0	0.48303	0.47236	0.46203	0.45204	0.44238	0.43303
60.0	0.48558	0.47533	0.46539	0.45575	0.44641	0.43736
65.0	0.48443	0.47469	0.46521	0.45600	0.44705	0.43836
70.0	0.47959	0.47043	0.46149	0.45277	0.44428	0.43602
75.0	0.47111	0.46259	0.45425	0.44610	0.43814	0.43037
80.0	0.45904	0.45123	0.44356	0.43603	0.42866	0.42144
85.0	0.44347	0.43643	0.42949	0.42264	0.41591	0.40930
90.0	0.42453	0.41832	0.41215	0.40604	0.40000	0.39404
95.0	0.40236	0.39702	0.39168	0.38635	0.38105	0.37579
100.0	0.37713	0.37270	0.36822	0.36371	0.35920	0.35468
105.0	0.34902	0.34554	0.34196	0.33831	0.33461	0.33087
110.0	0.31826	0.31575	0.31310	0.31034	0.30747	0.30454
115.0	0.28508	0.28356	0.28186	0.28000	0.27800	0.27589
120.0	0.24973	0.24922	0.24847	0.24753	0.24641	0.24514
125.0	0.21248	0.21297	0.21319	0.21318	0.21295	0.21253
130.0	0.17361	0.17510	0.17629	0.17720	0.17786	0.17830
135.0	0.13342	0.13591	0.13805	0.13988	0.14142	0.14271
140.0	0.09221	0.09567	0.09875	0.10149	0.10391	0.10604
145.0	0.05030	0.05471	0.05871	0.06233	0.06560	0.06856
150.0	0.00801	0.01334	0.01822	0.02269	0.02680	0.03056
155.0	-0.03434	-0.02814	-0.02241	-0.01712	-0.01221	-0.00768
160.0	-0.07643	-0.06941	-0.06287	-0.05679	-0.05113	-0.04585
165.0	-0.11794	-0.11014	-0.10286	-0.09604	-0.08966	-0.08368
170.0	-0.15855	-0.15004	-0.14206	-0.13455	-0.12750	-0.12087

TABLE IV. R OF GAMMA AND THETA.

	GAMMA					
THETA	0.05	0.10	0.15	0.20	0.25	0.30
10.0	0.12410	0.07442	0.02536	-0.02242	-0.06829	-0.11174
15.0	0.21000	0.16062	0.11142	0.06311	0.01632	-0.02840
20.0	0.29430	0.24559	0.19664	0.14816	0.10080	0.05515
25.0	0.37636	0.32870	0.28036	0.23207	0.18451	0.13828
30.0	0.45556	0.40930	0.36195	0.31423	0.26682	0.22036
35.0	0.53129	0.48679	0.44079	0.39399	0.34709	0.30076
40.0	0.60298	0.56058	0.51626	0.47075	0.42473	0.37888
45.0	0.67008	0.63010	0.58781	0.54393	0.49913	0.45411
50.0	0.73207	0.69482	0.65489	0.61297	0.56974	0.52588
55.0	0.78850	0.75425	0.71698	0.67734	0.63601	0.59365
60.0	0.83893	0.80795	0.77362	0.73656	0.69744	0.65690
65.0	0.88297	0.85549	0.82437	0.79018	0.75356	0.71516
70.0	0.92029	0.89653	0.86884	0.83778	0.80394	0.76797
75.0	0.95061	0.93074	0.90670	0.87900	0.84821	0.81494
80.0	0.97369	0.95786	0.93766	0.91354	0.88602	0.85570
85.0	0.98936	0.97770	0.96149	0.94112	0.91709	0.88995
90.0	0.99751	0.99010	0.97800	0.96154	0.94118	0.91743
95.0	0.99806	0.99496	0.98706	0.97464	0.95810	0.93793
100.0	0.99101	0.99225	0.98861	0.98032	0.96774	0.95129
105.0	0.97643	0.98199	0.98264	0.97855	0.97001	0.95740
110.0	0.95441	0.96425	0.96919	0.96932	0.96489	0.95624
115.0	0.92513	0.93918	0.94836	0.95272	0.95244	0.94779
120.0	0.88881	0.90696	0.92032	0.92887	0.93273	0.93213
125.0	0.84572	0.86783	0.88527	0.89795	0.90593	0.90938
130.0	0.79620	0.82210	0.84349	0.86020	0.87223	0.87971
135.0	0.74061	0.77012	0.79528	0.81589	0.83189	0.84334
140.0	0.67939	0.71227	0.74102	0.76538	0.78523	0.80055
145.0	0.61300	0.64900	0.68113	0.70905	0.73258	0.75167
150.0	0.54195	0.58080	0.61605	0.64732	0.67436	0.69707
155.0	0.46677	0.50817	0.54628	0.58066	0.61101	0.63717
160.0	0.38804	0.43168	0.47235	0.50958	0.54301	0.57241
165.0	0.30635	0.35190	0.39483	0.43462	0.47088	0.50330
170.0	0.22234	0.26944	0.31430	0.35636	0.39516	0.43036

TABLE V. I OF GAMMA AND THETA.

THETA	GAMMA 0.40	0.45	0.50	0.55	0.60	0.65
10.0	-0.18989	-0.22413	-0.25500	-0.28253	-0.30679	-0.32793
15.0	-0.10996	-0.14624	-0.17932	-0.20917	-0.23584	-0.25943
20.0	-0.02919	-0.06723	-0.10226	-0.13421	-0.16309	-0.18895
25.0	0.05181	0.01229	-0.02443	-0.05824	-0.08909	-0.11703
30.0	0.13240	0.09172	0.05359	0.01818	-0.01442	-0.04423
35.0	0.21200	0.17044	0.13120	0.09447	0.06036	0.02891
40.0	0.28997	0.24787	0.20781	0.17003	0.13468	0.10183
45.0	0.36574	0.32342	0.28284	0.24430	0.20797	0.17398
50.0	0.43873	0.39650	0.35572	0.31671	0.27968	0.24480
55.0	0.50838	0.46656	0.42589	0.38671	0.34927	0.31376
60.0	0.57416	0.53308	0.49282	0.45376	0.41619	0.38033
65.0	0.63557	0.59553	0.55600	0.51736	0.47995	0.44401
70.0	0.69214	0.65346	0.61495	0.57703	0.54006	0.50431
75.0	0.74345	0.70641	0.66921	0.63230	0.59605	0.56077
80.0	0.78909	0.75398	0.71839	0.68276	0.64751	0.61296
85.0	0.82873	0.79582	0.76209	0.72803	0.69404	0.66049
90.0	0.86207	0.83160	0.80000	0.76775	0.73529	0.70299
95.0	0.88884	0.86105	0.83182	0.80163	0.77095	0.74014
100.0	0.90885	0.88395	0.85730	0.82942	0.80073	0.77165
105.0	0.92194	0.90012	0.87627	0.85088	0.82442	0.79730
110.0	0.92802	0.90944	0.88856	0.86588	0.84184	0.81688
115.0	0.92703	0.91184	0.89409	0.87428	0.85285	0.83024
120.0	0.91899	0.90730	0.89282	0.87603	0.85737	0.83728
125.0	0.90395	0.89585	0.88475	0.87111	0.85537	0.83795
130.0	0.88204	0.87759	0.86995	0.85956	0.84685	0.83224
135.0	0.85341	0.85264	0.84853	0.84147	0.83189	0.82019
140.0	0.81828	0.82121	0.82065	0.81698	0.81060	0.80191
145.0	0.77693	0.78353	0.78652	0.78627	0.78314	0.77752
150.0	0.72967	0.73989	0.74641	0.74957	0.74972	0.74722
155.0	0.67685	0.69061	0.70062	0.70717	0.71059	0.71123
160.0	0.61888	0.63608	0.64950	0.65939	0.66606	0.66982
165.0	0.55620	0.57671	0.59343	0.60659	0.61645	0.62332
170.0	0.48929	0.51294	0.53284	0.54917	0.56216	0.57208

TABLE V. I OF GAMMA AND THETA.

	GAMMA					
THETA	0.75	0.80	0.85	0.90	0.95	1.00
10.0	-0.36157	-0.37451	-0.38516	-0.39375	-0.40048	-0.40558
15.0	-0.29800	-0.31337	-0.32640	-0.33730	-0.34629	-0.35355
20.0	-0.23216	-0.24984	-0.26515	-0.27829	-0.28945	-0.29884
25.0	-0.16455	-0.18441	-0.20188	-0.21716	-0.23042	-0.24185
30.0	-0.09569	-0.11757	-0.13708	-0.15438	-0.16963	-0.18301
35.0	-0.02610	-0.04984	-0.07124	-0.09042	-0.10755	-0.12279
40.0	0.04368	0.01826	-0.00485	-0.02578	-0.04465	-0.06163
45.0	0.11314	0.08623	0.06158	0.03907	0.01858	-0.00000
50.0	0.18173	0.15355	0.12753	0.10361	0.08168	0.06163
55.0	0.24894	0.21969	0.19252	0.16737	0.14415	0.12279
60.0	0.31426	0.28416	0.25604	0.22985	0.20553	0.18301
65.0	0.37718	0.34647	0.31761	0.29058	0.26535	0.24184
70.0	0.43723	0.40614	0.37676	0.34910	0.32314	0.29884
75.0	0.49396	0.46273	0.43305	0.40497	0.37847	0.35355
80.0	0.54693	0.51579	0.48604	0.45775	0.43093	0.40558
85.0	0.59573	0.56492	0.53533	0.50705	0.48010	0.45452
90.0	0.64000	0.60976	0.58055	0.55249	0.52562	0.50000
95.0	0.67940	0.64995	0.62135	0.59372	0.56714	0.54167
100.0	0.71363	0.68520	0.65742	0.63044	0.60435	0.57923
105.0	0.74243	0.71523	0.68849	0.66235	0.63695	0.61237
110.0	0.76557	0.73982	0.71432	0.68923	0.66471	0.64086
115.0	0.78289	0.75878	0.73471	0.71086	0.68741	0.66446
120.0	0.79426	0.77197	0.74951	0.72709	0.70487	0.68301
125.0	0.79957	0.77928	0.75860	0.73777	0.71698	0.69636
130.0	0.79881	0.78066	0.76192	0.74285	0.72362	0.70442
135.0	0.79196	0.77609	0.75945	0.74227	0.72476	0.70711
140.0	0.77909	0.76562	0.75119	0.73604	0.72038	0.70442
145.0	C.76028	0.74933	0.73722	0.72421	0.71052	0.69636
150.0	0.73569	0.72733	0.71763	0.70686	0.69526	0.68301
155.0	0.70550	0.69980	0.69259	0.68414	0.67470	0.66446
160.0	0.66995	0.66694	0.66227	0.65621	0.64900	0.64086
165.0	0.62929	0.62900	0.62691	0.62329	0.61837	0.61237
170.0	0.58385	0.58628	0.58679	0.58562	0.58303	0.57923

TABLE V. I OF GAMMA AND THETA.

	GAMMA					
THETA	1.10	1.15	1.20	1.25	1.30	1.35
10.0	-0.41160	-0.41287	-0.41316	-0.41263	-0.41138	-0.40951
15.0	-0.36366	-0.36684	-0.36897	-0.37018	-0.37059	-0.37030
20.0	-0.31296	-0.31803	-0.32197	-0.32492	-0.32698	-0.32828
25.0	-0.25987	-0.26680	-0.27252	-0.27718	-0.28089	-0.28375
30.0	-0.20481	-0.21353	-0.22100	-0.22733	-0.23265	-0.23707
35.0	-0.14819	-0.15864	-0.16779	-0.17575	-0.18265	-0.18858
40.0	-0.09043	-0.10255	-0.11331	-0.12284	-0.13125	-0.13866
45.0	-0.03200	-0.04567	-0.05796	-0.06899	-0.07886	-0.08768
50.0	0.02669	0.01156	-0.00217	-0.01461	-0.02587	-0.03604
55.0	0.08517	0.06869	0.05363	0.03988	0.02733	0.01588
60.0	0.14300	0.12531	0.10903	0.09406	0.08031	0.06768
65.0	0.19974	0.18097	0.16359	0.14753	0.13268	0.11896
70.0	0.25496	0.23525	0.21691	0.19987	0.18404	0.16934
75.0	0.30825	0.28774	0.26858	0.25069	0.23400	0.21843
80.0	0.35918	0.33805	0.31821	0.29961	0.28218	0.26586
85.0	0.40739	0.38578	0.36541	0.34624	0.32821	0.31126
90.0	0.45249	0.43057	0.40984	0.39024	0.37175	0.35430
95.0	0.49415	0.47209	0.45114	0.43127	0.41245	0.39464
100.0	0.53205	0.51001	0.48901	0.46902	0.45002	0.43197
105.0	0.56589	0.54405	0.52316	0.50320	0.48416	0.46602
110.0	0.59544	0.57396	0.55333	0.53355	0.51462	0.49652
115.0	0.62045	0.59949	0.57928	0.55984	0.54116	0.52324
120.0	0.64074	0.62046	0.60083	0.58186	0.56358	0.54598
125.0	0.65615	0.63671	0.61781	0.59946	0.58171	0.56456
130.0	0.66657	0.64812	0.63008	0.61250	0.59542	0.57885
135.0	0.67191	0.65459	0.63756	0.62088	0.60459	0.58874
140.0	0.67214	0.65608	0.64018	0.62453	0.60916	0.59414
145.0	0.66726	0.65257	0.63794	0.62342	0.60910	0.59502
150.0	0.65730	0.64410	0.63083	0.61757	0.60440	0.59137
155.0	0.64233	0.63073	0.61893	0.60703	0.59510	0.58322
160.0	0.62248	0.61256	0.60232	0.59186	0.58127	0.57063
165.0	0.59789	0.58973	0.58112	0.57219	0.56302	0.55370
170.0	0.56875	0.56240	0.55550	0.54816	0.54048	0.53256

TABLE V. I OF GAMMA AND THETA.

	GAMMA					
THETA	1.45	1.50	1.55	1.60	1.65	1.70
10.0	-0.40429	-0.40110	-0.39759	-0.39383	-0.38987	-0.38574
15.0	-0.36802	-0.36618	-0.36396	-0.36142	-0.35862	-0.35559
20.0	-0.32894	-0.32847	-0.32755	-0.32626	-0.32464	-0.32274
25.0	-0.28736	-0.28826	-0.28866	-0.28862	-0.28819	-0.28743
30.0	-0.24359	-0.24586	-0.24756	-0.24878	-0.24955	-0.24993
35.0	-0.19797	-0.20158	-0.20459	-0.20704	-0.20901	-0.21054
40.0	-0.15084	-0.15578	-0.16005	-0.16373	-0.16687	-0.16953
45.0	-0.10256	-0.10878	-0.11430	-0.11917	-0.12347	-0.12724
50.0	-0.05350	-0.06096	-0.06768	-0.07371	-0.07913	-0.08398
55.0	-0.00404	-0.01268	-0.02054	-0.02769	-0.03418	-0.04008
60.0	0.04546	0.03570	0.02675	0.01855	0.01102	0.00412
65.0	0.09461	0.08381	0.07384	0.06464	0.05614	0.04829
70.0	0.14303	0.13128	0.12037	0.11024	0.10084	0.09210
75.0	0.19038	0.17775	0.16598	0.15501	0.14476	0.13520
80.0	0.23627	0.22287	0.21033	0.19859	0.18759	0.17728
85.0	0.28036	0.26630	0.25308	0.24066	0.22898	0.21800
90.0	0.32232	0.30769	0.29390	0.28090	0.26864	0.25707
95.0	0.36183	0.34675	0.33249	0.31900	0.30625	0.29418
100.0	0.39858	0.38316	0.36854	0.35468	0.34153	0.32905
105.0	0.43230	0.41666	0.40179	0.38765	0.37421	0.36142
110.0	0.46273	0.44699	0.43198	0.41768	0.40404	0.39104
115.0	0.48964	0.47392	0.45889	0.44452	0.43080	0.41768
120.0	0.51282	0.49724	0.48230	0.46799	0.45427	0.44114
125.0	0.53210	0.51678	0.50204	0.48789	0.47429	0.46124
130.0	0.54733	0.53238	0.51796	0.50408	0.49071	0.47784
135.0	0.55839	0.54393	0.52994	0.51643	0.50338	0.49080
140.0	0.56521	0.55134	0.53789	0.52485	0.51223	0.50002
145.0	0.56772	0.55456	0.54174	0.52928	0.51718	0.50544
150.0	0.56591	0.55355	0.54147	0.52968	0.51819	0.50701
155.0	0.55980	0.54833	0.53708	0.52604	0.51525	0.50472
160.0	0.54942	0.53894	0.52860	0.51841	0.50840	0.49859
165.0	0.53486	0.52545	0.51609	0.50683	0.49768	0.48866
170.0	0.51624	0.50796	0.49966	0.49139	0.48317	0.47502

TABLE V. I OF GAMMA AND THETA.

	GAMMA					
THETA	1.80	1.85	1.90	1.95	2.00	2.05
10.0	-0.37713	-0.37270	-0.36822	-0.36371	-0.35919	-0.35468
15.0	-0.34902	-0.34554	-0.34196	-0.33831	-0.33461	-0.33087
20.0	-0.31826	-0.31575	-0.31310	-0.31034	-0.30747	-0.30454
25.0	-0.28508	-0.28356	-0.28186	-0.28000	-0.27800	-0.27589
30.0	-0.24973	-0.24921	-0.24847	-0.24753	-0.24641	-0.24514
35.0	-0.21248	-0.21297	-0.21319	-0.21317	-0.21295	-0.21253
40.0	-0.17361	-0.17510	-0.17629	-0.17720	-0.17786	-0.17830
45.0	-0.13342	-0.13590	-0.13805	-0.13988	-0.14142	-0.14271
50.0	-0.09221	-0.09567	-0.09875	-0.10149	-0.10391	-0.10604
55.0	-0.05030	-0.05471	-0.05871	-0.06233	-0.06560	-0.06856
60.0	-0.00801	-0.01333	-0.01822	-0.02269	-0.02679	-0.03056
65.0	0.03434	0.02814	0.02242	0.01712	0.01222	0.00768
70.0	0.07643	0.06941	0.06288	0.05679	0.05113	0.04585
75.0	0.11794	0.11015	0.10286	0.09604	0.08966	0.08368
80.0	0.15855	0.15004	0.14206	0.13455	0.12750	0.12087
85.0	0.19795	0.18880	0.18017	0.17205	0.16438	0.15714
90.0	0.23585	0.22612	0.21692	0.20823	0.20000	0.19222
95.0	0.27195	0.26172	0.25202	0.24282	0.23410	0.22583
100.0	0.30599	0.29532	0.28519	0.27557	0.26642	0.25772
105.0	0.33769	0.32668	0.31620	0.30622	0.29671	0.28765
110.0	0.36682	0.35555	0.34480	0.33454	0.32475	0.31540
115.0	0.39317	0.38172	0.37078	0.36032	0.35031	0.34074
120.0	0.41652	0.40498	0.39393	0.38335	0.37321	0.36349
125.0	0.43670	0.42516	0.41409	0.40346	0.39326	0.38347
130.0	0.45355	0.44211	0.43110	0.42051	0.41033	0.40053
135.0	0.46696	0.45568	0.44482	0.43435	0.42427	0.41455
140.0	0.47681	0.46580	0.45516	0.44489	0.43498	0.42541
145.0	0.48303	0.47236	0.46203	0.45204	0.44238	0.43303
150.0	0.48558	0.47533	0.46539	0.45575	0.44641	0.43736
155.0	0.48443	0.47469	0.46521	0.45600	0.44705	0.43836
160.0	0.47959	0.47043	0.46149	0.45277	0.44428	0.43602
165.0	0.47111	0.46259	0.45425	0.44610	0.43814	0.43037
170.0	0.45904	0.45123	0.44356	0.43603	0.42866	0.42144

TABLE V. I OF GAMMA AND THETA.

P	R = 40.00	R = 20.00	R = 10.00	R = 5.00
1.0	1.00000	1.00000	1.00000	1.00000
1.2	0.99912	0.99815	0.99588	0.98958
1.4	0.99699	0.99367	0.98589	0.96429
1.6	0.99408	0.98753	0.97222	0.92969
1.8	0.99065	0.98030	0.95610	0.88889
2.0	0.98685	0.97230	0.93827	0.84375
2.2	0.98279	0.96374	0.91919	0.79545
2.4	0.97852	0.95476	0.89918	0.74479
2.6	0.97411	0.94545	0.87844	0.69231
2.8	0.96957	0.93589	0.85714	0.63839
3.0	0.96493	0.92613	0.83539	0.58333
3.2	0.96022	0.91620	0.81327	0.52734
3.4	0.95545	0.90614	0.79085	0.47059
3.6	0.95062	0.89597	0.76818	0.41319
3.8	0.94574	0.88570	0.74529	0.35526
4.0	0.94083	0.87535	0.72222	0.29688
4.2	0.93588	0.86493	0.69900	0.23810
4.4	0.93091	0.85444	0.67565	0.17898
4.6	0.92591	0.84391	0.65217	0.11957
4.8	0.92088	0.83333	0.62860	0.05990
5.0	0.91584	0.82271	0.60494	0.00000

TABLE VI. H OF P AND R.

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13. ABSTRACT

Through the utilization of the functional matrix description of linear integrated circuits, a design method is evolved from which the terminating admittances may be specified. These terminating admittances are optimized with respect to power gain for a stable device and optimized with respect to power gain for a specified stability level. The total input and output conductances are obtained for use in frequency response design. Finally the procedure is extended to include the case in which a not optimum conductance is forced on the designer.

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